1 Rational Numbers

Exercise 1.1

1. The positive rational numbers are : $\frac{3}{7}, \frac{-9}{11}, \frac{-11}{-22}$.

2. (a) $\frac{3}{7} + \frac{2}{5} = \frac{3 \times 5 + 2 \times 7}{35} = \frac{15 + 14}{35} = \frac{29}{35}$. [LCM of 5 and 7 is 35.]

   (b) $\frac{9}{20} + \frac{-3}{8} = \frac{9 \times 2 - 3 \times 5}{40} = \frac{18 - 15}{40} = \frac{3}{40}$. [LCM of 20 and 8 is 40.]

   (c) $\frac{-2}{5} + \frac{-3}{20} = \frac{-2 \times 4 - 3}{20} = \frac{-8 - 3}{20} = \frac{-11}{20}$. [LCM of 5 and 20 is 20.]

   (d) $\frac{4}{9} + \frac{7}{-15} = \frac{4}{9} - \frac{7}{15} = \frac{4 \times 5 - 7 \times 3}{45} = \frac{20 - 21}{45} = \frac{-1}{45}$. [LCM of 9 and 15 is 15.]

3. (a) $\frac{4}{27} - \frac{7}{9} = \frac{4 - 7 \times 3}{27} = \frac{4 - 21}{27} = \frac{17}{27}$. [LCM of 9 and 27 is 27.]

   (b) $\frac{6}{13} - \frac{2}{5} = \frac{6 \times 5 - 2 \times 13}{65} = \frac{30 - 26}{65} = \frac{4}{65}$. [LCM of 13 and 5 is 65.]

   (c) $\frac{8}{15} - \frac{17}{20} = \frac{8 \times 4 - 17 \times 3}{60} = \frac{32 - 51}{60} = \frac{-19}{60}$. [LCM of 15 and 20 is 60.]

   (d) $\frac{6}{11} - \frac{2}{5} = \frac{6 \times 5 - 2 \times 11}{55} = \frac{30 - 22}{55} = \frac{8}{55}$. [LCM of 11 and 5 is 55.]

4. (a) $\frac{3}{7} \times \frac{-2}{5} = \frac{-6}{35}$.

   (b) $\frac{6}{\cancel{-11}_{-1}} \times \frac{\cancel{22}^{2}}{35} = \frac{12}{-35} = \frac{-12}{35}$.

   (c) $\frac{\cancel{-11}^{-1}}{24} \times \frac{-13}{\cancel{22}_{2}} = \frac{13}{24 \times 2} = \frac{13}{48}$.

   (d) $\frac{-6}{\cancel{23}_{1}} \times \frac{\cancel{-46}^{-2}}{-5} = \frac{12}{-5} = -2\frac{2}{5}$.

5. (a) $\frac{4}{5} \surd \frac{3}{10} = \frac{4}{\cancel{5}_{1}} \times \frac{\cancel{10}^{2}}{3} = \frac{8}{3} = 2\frac{2}{3}$.

   (b) $\frac{7}{15} \div \frac{-9}{14} = \frac{7}{15} \times \frac{14}{-9} = \frac{98}{-135} = \frac{-98}{135}$.

   (c) $\frac{2}{-11} \div \frac{4}{-7} = \frac{\cancel{2}^{1}}{-11} \times \frac{-7}{\cancel{4}_{2}} = \frac{-7}{-22} = \frac{7}{22}$.

   (d) $\frac{12}{-20} \div \frac{3}{11} = \frac{\cancel{-12}^{-4}}{20} \times \frac{11}{\cancel{3}_{1}} = \frac{\cancel{-44}^{-11}}{\cancel{20}_{5}} = \frac{-11}{5}$.

6. The absolute value of :

   (a) $\left|\frac{-5}{11}\right| = \frac{5}{11}$.

   (b) $\left|\frac{7}{12}\right| = \frac{7}{12}$.

   (c) $\left|\frac{\cancel{-2}^{-1}}{7} \times \frac{3}{\cancel{8}_{4}}\right| = \left|\frac{-3}{28}\right| = \frac{3}{28}$.

(d) $\left|\frac{3}{11} - \frac{2}{9}\right| = \left|\frac{3 \times 9 - 2 \times 11}{99}\right| = \left|\frac{27 - 22}{99}\right| = \left|\frac{5}{99}\right| = \frac{5}{99}$.

(e) $\left|\frac{2}{5} + \frac{-3}{4}\right| = \left|\frac{2}{5} - \frac{3}{4}\right| = \left|\frac{2 \times 4 - 3 \times 5}{20}\right| = \left|\frac{8 - 15}{20}\right| = \left|\frac{-7}{20}\right| = \frac{-7}{20}$.

7. (a) $\frac{-20}{25} = \frac{-20 \div 5}{25 \div 5} = \frac{-4}{5}$. [HCF of 20 and 25 = 5]

(b) $\frac{36}{-72} = \frac{36 \div 36}{-72 \div 36} = \frac{1}{-2} = \frac{-1}{2}$. [HCF of 36 and 72 = 36]

(c) $\frac{-18}{-20} = \frac{18}{20} = \frac{18 \sqrt{} 2}{20 \sqrt{} 2} = \frac{9}{20}$. [HCF of 18 and 20 = 2]

(d) $\frac{6}{40} = \frac{6 \sqrt{} 2}{40 \sqrt{} 2} = \frac{3}{20}$. [HCF of 6 and 20 = 2]

(e) $\frac{14}{-23} = \frac{14 \div -1}{-23 \div -1} = \frac{-14}{23}$. [Making the denominator positive]

Exercise 1.2

1. (a) Commutative property of addition (b) Additive inverse property
(c) Zero property of addition (d) Association property of addition.

2. We have to verify that x + y = y + x.

(a) Given : $x = \frac{-4}{9}$ and $y = \frac{-2}{5}$.

$$\text{LHS} = x + y = \frac{-4}{9} + \frac{-2}{5} = \frac{(-4 \times 5) + (-2 \times 9)}{45} = \frac{-20 - 18}{45} = \frac{-38}{45}.$$

$$\text{RHS} = y + x = \frac{-2}{5} + \frac{-4}{9} = \frac{(-2 \times 9) + (-4 \times 5)}{45} = \frac{-18 - 20}{45} = \frac{-38}{45}.$$

∴ LHS = RHS. Hence, verified.

(b) Given : $x = \frac{-3}{8}$ and $y = \frac{-5}{6}$.

$$x + y = \frac{-3}{8} + \frac{-5}{6} = \frac{(-3 \times 3) + (-5 \times 4)}{24} = \frac{-9 - 20}{24} = \frac{-29}{24}.$$

$$y + x = \frac{-5}{6} + \frac{-3}{8} = \frac{(-5 \times 4) + (-3 \times 3)}{24} = \frac{-20 - 9}{24} = \frac{-29}{24}.$$

∴ x + y = y + x. Hence, verified.

3. Closure property of addition : For any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b} + \frac{c}{d}$ is a rational number.

(a) $\frac{-3}{7} + \frac{-4}{11} = \frac{-3}{7} - \frac{4}{11} = \frac{-3 \times 11 - 4 \times 7}{77} = \frac{-33 - 28}{77} = \frac{-61}{77}$, which is a rational numbers.
Hence, verified.

(b) $\frac{-9}{14}+\frac{1}{7}=\frac{-9+1\times2}{14}=\frac{-9+2}{14}=\frac{-7}{14}=\frac{-1}{2}$, which is a rational number. Hence, verified.

4. Closure property of subtraction : For any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b}-\frac{c}{d}$ is a rational number.

(a) $\frac{-2}{3}-\frac{5}{9}=\frac{-2\times3-5}{9}=\frac{-6-5}{9}=\frac{-11}{9}$, which is a rational number. [LCM of 3 and 9 = 9]

(b) $\frac{-3}{11}-\frac{-7}{22}=\frac{-3\times2+7}{22}=\frac{-6+7}{22}=\frac{1}{22}$, which is a rational number. Hence, verified.

5. (a) Given : $x=\frac{-5}{7}$

$\therefore -(-x)=-\left(-\frac{-5}{7}\right)=-\left(\frac{5}{7}\right)=-\frac{5}{7}=x.$

Hence, verified.

(b) Given : $x=\frac{9}{4}$

$\therefore -(-x)=-\left(-\frac{9}{4}\right)=\frac{9}{4}=x.$

Hence, verified.

6. (a) Given : $x=\frac{-5}{11}$ and $y=\frac{-2}{3}$

$\therefore\ x-y=\frac{-5}{11}-\left(\frac{-2}{3}\right)=\frac{-5}{11}+\frac{2}{3}=\frac{-5\times3+2\times11}{33}=\frac{-15+22}{33}=\frac{7}{33}.$ [LCM of 3 and 11 = 33]

$y-x=\frac{-2}{3}-\left(\frac{-5}{11}\right)=\frac{-2}{3}+\frac{5}{11}=\frac{-2\times11+5\times3}{33}=\frac{-22+15}{33}=\frac{-7}{33}.$

Thus, $x-y\neq y-x$.

(b) Given : $x=\frac{5}{14}$ and $y=\frac{3}{7}$

$\therefore\ x-y=\frac{5}{14}-\frac{3}{7}=\frac{5-3\times2}{14}=\frac{5-6}{14}=\frac{-1}{14}.$ [LCM of 7 and 14 = 14]

$y-x=\frac{3}{7}-\frac{5}{14}=\frac{3\times2-5}{14}=\frac{6-5}{14}=\frac{-1}{14}.$

Thus, $x-y\neq y-x$.

7. (a) We have $\frac{-3}{8}$, $\frac{-2}{3}$ and $\frac{5}{8}$.

$\left(\frac{-3}{8}+\frac{-2}{3}\right)+\frac{5}{8}=\left(\frac{-3\times3+(-2\times8)}{24}\right)+\frac{5}{8}$

$=\frac{-9-16}{24}+\frac{5}{8}=\frac{-25}{24}+\frac{5}{8}=\frac{-25+5\times3}{24}=\frac{-25+15}{24}=\frac{-10}{24}.$ ...(i)

Now, $\frac{-3}{8} + \left(\frac{-2}{3} + \frac{5}{8}\right) = \frac{-3}{8} + \left(\frac{-2 \times 8 + 5 \times 3}{24}\right) = \frac{-3}{8} + \left(\frac{-16 + 15}{24}\right)$ [LCM of 3 and 8 = 24]

$= \frac{-3}{8} - \frac{1}{24} = \frac{-3 \times 3 - 1}{24} = \frac{-9 - 1}{24} = \frac{-10}{24}$. ...(ii)

From equations (i) and (ii), we find that both the sums are equal.
Hence, the associative property of addition is verified.

(b) We have $\frac{7}{12}, \frac{-5}{6}$ and $\frac{-1}{4}$.

First, we take : $\left(\frac{7}{12} + \frac{-5}{6}\right) + \frac{-1}{4}$.

$\left(\frac{7}{12} - \frac{5}{6}\right) - \frac{1}{4} = \left(\frac{7 - 5 \times 2}{12}\right) - \frac{1}{4} = \frac{7 - 10}{12} - \frac{1}{4} = \frac{-3}{12} - \frac{1}{4} = \frac{-1}{4} - \frac{1}{4} = \frac{-2}{4} = \frac{-1}{2}$. ...(i)

Now, $\frac{7}{12} + \left(\frac{-5}{6} + \frac{-1}{4}\right) = \frac{7}{12} + \left(\frac{-5 \times 2 - 1 \times 3}{12}\right)$

$= \frac{7}{12} + \left(\frac{-10 - 3}{12}\right) = \frac{7}{12} - \frac{13}{12} = \frac{-6}{12} = \frac{-1}{2}$. ...(ii)

From equations (i) and (ii), we find that both the sums are equal.
Hence, the associative property of addition is verified.

8. (a) $\frac{-11}{16} + \frac{-2}{5} + \frac{1}{8} = \frac{-11}{16} - \frac{2}{5} + \frac{1}{8} = \frac{-11 \times 5}{16 \times 5} - \frac{2 \times 16}{5 \times 16} + \frac{1 \times 10}{8 \times 10}$ [LCM of 5, 8 and 16 = 80]

$\frac{-55}{80} - \frac{32}{80} + \frac{10}{80} = \frac{-87}{80} - \frac{10}{80} = \frac{-77}{80}$.

(b) $\frac{-7}{12} + 4 + \frac{-2}{9} + \frac{5}{6} = \frac{-7}{12} + \frac{4}{1} - \frac{2}{9} + \frac{5}{6} = \frac{-7 \times 3}{12 \times 3} + \frac{4 \times 36}{1 \times 36} - \frac{2 \times 4}{9 \times 4} + \frac{5 \times 6}{6 \times 6}$

$= \frac{-21}{36} + \frac{144}{36} - \frac{8}{36} + \frac{30}{36} = \frac{-21 + 155 - 8 + 30}{36} = \frac{145}{36}$.

9. To get the answer, we will subtract $\frac{16}{25}$ from $\frac{-4}{5}$.

$\therefore$ Required number $= \frac{-4}{5} - \frac{16}{25} = \frac{-4 \times 5}{5 \times 5} - \frac{16}{25} = \frac{-20}{25} - \frac{16}{25} = \frac{-36}{25}$.

10. Required number = Product ÷ One number $= \frac{25}{34} \div \frac{-5}{17} = \frac{25}{\cancel{34}_2} \times \frac{\cancel{17}^1}{-5} = \frac{\cancel{25}^5}{\cancel{-10}_2} = \frac{5}{-2} = \frac{-5}{2}$.

Exercise 1.3

1. The additive inverse of :

(a) $\frac{2}{3} = \frac{-2}{3}$ (b) $-15 = 15$ (c) $\frac{-6}{7} = \frac{6}{7}$ (d) $\frac{9}{-2} = \frac{9}{2}$

(e) $0 = 0$ (f) $\frac{-9}{-11} = \frac{-9}{11}$

2. Multiplicative inverse of :

(a) -4 is $\frac{1}{-4}$ (b) $\frac{3}{8}$ is $\frac{8}{3}$ (c) 8 is $\frac{1}{8}$ (d) $-2 \times \frac{3}{5} = \frac{-6}{5}$ is $\frac{5}{-6}$

(e) $\frac{\cancel{2}^{1}}{-7} \times \frac{3}{\cancel{4}_{2}} = \frac{3}{-14}$ is $\frac{-14}{3}$ (f) $\cancel{-7} \times \frac{1}{\cancel{-7}} = 1$ is 1.

3. (a) We have $\frac{-6}{5} + \frac{3}{4} = \frac{3}{4} + \left(\frac{-6}{5}\right)$

$$\text{LHS} = \frac{-6}{5} + \frac{3}{4} = \frac{-6 \times 4}{5 \times 4} + \frac{3 \times 5}{4 \times 5} = \frac{-24}{20} + \frac{15}{20} = \frac{-9}{20}. \quad \text{...(i)} \quad \text{[LCM of 4 and 5 = 20]}$$

$$\text{RHS} = \frac{3}{4} + \left(\frac{-6}{5}\right) = \frac{3}{4} - \frac{6}{5} = \frac{3 \times 5}{4 \times 5} - \frac{6 \times 4}{5 \times 4} = \frac{15}{20} - \frac{24}{20} = \frac{-9}{20}. \quad \text{...(ii)}$$

From equations (i) and (ii), LHS = RHS.
Hence, verified.

(b) We have $\frac{2}{-5} + \frac{6}{-7} = \frac{6}{-7} + \frac{2}{-5}$

$$\text{LHS} = \frac{-2}{5} - \frac{6}{7} = \frac{-2 \times 7}{5 \times 7} - \frac{6 \times 5}{7 \times 5} = \frac{-14}{35} - \frac{30}{35} = \frac{-44}{35}. \quad \text{...(i) [LCM of 5 and 7 = 35]}$$

$$\text{RHS} = \frac{6}{-7} + \frac{2}{-5} = \frac{-6}{7} - \frac{2}{5} = \frac{-6 \times 5}{7 \times 5} - \frac{2 \times 7}{5 \times 7} = \frac{-30}{35} - \frac{14}{35} = \frac{-44}{35}. \quad \text{...(ii)}$$

From equations (i) and (ii), LHS = RHS.
Hence, verified.

(c) We have $\frac{2}{7} + \left(\frac{3}{4} + \frac{1}{2}\right) = \left(\frac{2}{7} + \frac{3}{4}\right) + \frac{1}{2}.$

$$\text{LHS} = \frac{2}{7} + \left(\frac{3}{4} + \frac{1}{2}\right) = \frac{2}{7} + \left(\frac{3+2}{4}\right) = \frac{2}{7} + \frac{5}{4} = \frac{8+35}{28} = \frac{43}{28}. \quad \text{...(i)}$$

$$\text{RHS} = \left(\frac{2}{7} + \frac{3}{4}\right) + \frac{1}{2} = \left(\frac{8+21}{28}\right) + \frac{1}{2} = \frac{29}{28} + \frac{1}{2} = \frac{29+14}{28} = \frac{43}{28}. \quad \text{...(ii)}$$

From equations (i) and (ii), LHS = RHS.
Hence, verified.

(d) We have $\left(-4 + \frac{-3}{5}\right) + \frac{-1}{4} = -4 + \left(\frac{-3}{5} + \frac{-1}{4}\right)$

$$\text{LHS} = \left(-4 + \frac{-3}{5}\right) + \frac{-1}{4} = \left(\frac{-1}{4} - \frac{3}{5}\right) - \frac{1}{4}$$

$$= \left(\frac{-4 \times 5 - 3}{5}\right) - \frac{1}{4} = \frac{-23}{5} - \frac{1}{4} = \frac{-23 \times 4 - 1 \times 5}{20} = \frac{-92 - 5}{20} = \frac{-97}{20}. \quad \text{...(i)}$$

$$\text{RHS} = -4 + \left(\frac{-3}{5} + \frac{-1}{4}\right) = -4 + \left(\frac{-3 \times 4 - 1 \times 5}{20}\right)$$

$$= -4 + \left(\frac{-12-5}{20}\right) = -4 - \frac{17}{20} = \frac{-4 \times 20 - 17}{20} = \frac{-80-17}{20} = \frac{-97}{20} \qquad \text{...(ii)}$$

From equations (i) and (ii), LHS = RHS. Hence, verified.

4. (a) Closure property of multiplication
 (b) Distributive property of multiplication over addition
 (c) Property of multiplicative inverse
 (d) Property of multiplicative identity
 (e) Property of zero

5. We have to verify that $a \times (b \times c) = (a \times b) \times c$

 (a) Given $a = \frac{-10}{11}$, $b = -\frac{2}{3}$ and $c = \frac{4}{5}$.

 $$\therefore\ a \times (b \times c) = \frac{-10}{11} \times \left(\frac{-2}{3} \times \frac{4}{5}\right) = \frac{-10}{11} \times \frac{-8}{15} = \frac{80}{165}.$$

 $$(a \times b) \times c = \left(\frac{-10}{11} \times \frac{-2}{3}\right) \times \frac{4}{5} = \frac{20}{33} \times \frac{4}{5} = \frac{80}{165}.$$

 Both the products are equal. Hence, verified.

 (b) Given : $a = \frac{5}{12}$, $b = \frac{-8}{3}$ and $c = \frac{9}{4}$.

 $$\therefore\ a \times (b \times c) = \frac{5}{12} \times \left(\frac{-\cancel{8}^{-2}}{\cancel{3}_1} \times \frac{\cancel{9}^{3}}{\cancel{4}_1}\right) = \frac{5}{\cancel{12}_2} \times \frac{-\cancel{6}^{1}}{1} = \frac{-5}{2}.$$

 $$(a \times b) \times c = \left(\frac{5}{\cancel{12}_3} \times \frac{-\cancel{8}^{2}}{3}\right) \times \frac{9}{4} = \frac{-10}{\cancel{9}} \times \frac{\cancel{9}}{4} = \frac{-\cancel{10}^{5}}{\cancel{4}_2} = \frac{-5}{2}.$$

Both the products are equal. Hence, verified.

6. We have to verify that $x \div y \neq y \div x$

 (a) Given $x = 15$ and $y = \frac{5}{8}$

 $$x \div y = 15 \div \frac{5}{8} = \cancel{15}^{3} \times \frac{8}{\cancel{5}_1} = \frac{8}{3}.$$

 $$y \div x = \frac{5}{8} \div 15 = \frac{\cancel{5}^{1}}{8} \times \frac{1}{\cancel{15}_3} = \frac{1}{24}.$$

 Both the answers are different. Hence, verified.

 (b) Given $x = \frac{21}{25}$ and $y = \frac{4}{5}$.

 $$x \div y = \frac{21}{25} \surd \frac{4}{5} = \frac{21}{\cancel{25}_5} \times \frac{\cancel{5}^{1}}{4} = \frac{21}{20}.$$

 $$y \div x = \frac{4}{5} \surd \frac{21}{25} = \frac{4}{\cancel{5}_1} \times \frac{\cancel{25}^{6}}{21} = \frac{20}{21}.$$

Both the answers are different. Hence, verified.

7. We have $x = \frac{-5}{9}, y = \frac{3}{10}$ and $z = \frac{-2}{15}$

$$\therefore x \times (y + z) = xy + xz = \frac{-5}{9} \times \frac{3}{10} + \frac{-5}{9} \times \frac{-2}{15}$$

$$= \frac{-15}{90} + \frac{10}{135} = \frac{-1}{6} + \frac{2}{27} = \frac{-1 \times 9}{6 \times 9} + \frac{2 \times 2}{27 \times 2} = \frac{-9}{54} + \frac{4}{54} = \frac{-5}{54}.$$

8. (a) $\left(\frac{2}{9}\right)^{-1} = \frac{1}{2/9} = \frac{9}{2}.$ (b) $(-8)^{-1} = \frac{1}{-8} = \frac{-1}{8}.$

(c) $\left(\frac{4}{-5}\right)^{-1} = \frac{1}{4/-5} = \frac{-5}{4}.$ (d) $\left(\frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{-3}^{-1}}{\cancel{4}_2}\right)^{-1} = \left(\frac{-1}{2}\right)^{-1} = \frac{1}{-1/2} = \frac{2}{-1} = -2.$

9. (a) $\frac{3}{8} \surd \frac{2}{5} = \frac{3}{8} \times \frac{5}{2} = \frac{15}{16}.$ (b) $-16 \div \frac{2}{5} = \frac{-16}{1} \times \frac{5}{2} = \frac{-80}{2} = -40.$

(c) $\frac{-7}{8} \div \left(\frac{2}{5} \div \frac{6}{5}\right) = \frac{-7}{8} \div \left(\frac{2}{\cancel{5}} \times \frac{\cancel{5}}{6}\right) = \frac{-7}{8} \div \frac{2}{6} = \frac{-7}{8} \times \frac{\cancel{6}^3}{\cancel{2}_1} = \frac{-7 \times 3}{8} = \frac{-21}{8}.$

(d) $\frac{-9}{17} \div \frac{54}{51} = \frac{\cancel{-9}^{-1}}{\cancel{17}_1} \times \frac{\cancel{51}^3}{\cancel{54}_6} = \frac{-3}{6}.$

10. (a) $\frac{-7}{2} \times \frac{\cancel{3}^1}{14} \times \frac{-8}{\cancel{15}_5} = \frac{-7 \times (-8)}{2 \times 14 \times 5} = \frac{\cancel{56}^2}{\cancel{28}_1 \times 5} = \frac{2}{5}.$

(b) $\frac{5}{-12} \times \frac{8}{-18} \times -20 = \frac{5}{-12} \times \frac{-8}{18} \times \frac{-20}{1} = \frac{\cancel{40}\,\cancel{10}^5 \times (-20)}{\cancel{12}_3 \times \cancel{18}_9} = \frac{-100}{27}.$

Exercise 1.4

1. (a) We have $\frac{-4}{9}$ .... $\frac{4}{-9}$

By cross multiplication,

$\frac{-4}{9} \quad \frac{4}{-9}$

$-4 \times (-9)$ $\quad 4 \times 9$

36 $\quad$ 36

Thus, $\frac{-4}{9} = \frac{4}{-9}.$

(b) We have $\frac{6}{15}$ .... $\frac{-2}{5}$

By cross multiplication,

$\frac{6}{15} \quad \frac{-2}{5}$

6×5 $\quad -2 \times 15$

30 $\quad$ –30

Thus, $\frac{6}{15} > \frac{-2}{5}$

(c) We have $\frac{8}{9}$ .... $\frac{6}{5}$

By cross multiplication,

$$\frac{8}{9} \times \frac{6}{5}$$

8×5 $\quad$ 6×9

40 $\quad$ 50

Thus, $\frac{8}{9} < \frac{6}{5}$.

(d) We have $\frac{-2}{5}$ .... $\frac{-3}{7}$

By cross multiplication,

$$\frac{-2}{5} \times \frac{-3}{7}$$

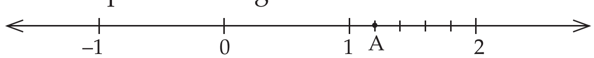
-2×7 $\quad$ -3×5

-14 $\quad$ -15

Thus, $\frac{-2}{5} > \frac{-3}{7}$

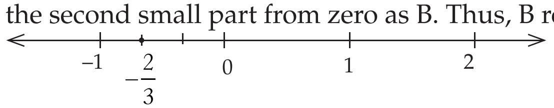
2. (a) $\frac{6}{5} = 1\frac{1}{5}$. To represent $\frac{6}{5}$ on the number line, follow these steps.

- Draw a number line.
- Take 1 complete unit to the right of zero.
- Divide the unit length between 1 and 2 into five equal parts.
- Take one small part to the right of 1 and mark it as A.



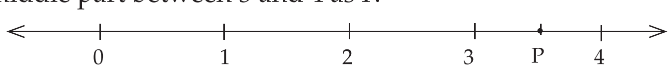
Thus, A represents $\frac{6}{5}$.

(b) To represent $-\frac{2}{3}$ on the number line, divide the unit length between 0 and –1 into three equal parts. Mark the second small part from zero as B. Thus, B represent $-\frac{2}{3}$.



(c) To represent $3\frac{1}{2}$ on the number line:

- Take three full units.
- Divide the unit length between 3 and 4 into two equal parts.
- Mark the middle part between 3 and 4 as P.



Thus, P represents $3\frac{1}{2}$.

(d) Similar work to be done as (c) above.

3. (a) We have $\frac{4}{5}, \frac{-1}{2}, \frac{3}{6}$ and $\frac{3}{4}$.

LCM of the denominators 5, 2, 6 and 4 is 60. Converting the given rational numbers into equivalent rational numbers with denominator 60, we get

2	5	2	6	4
2	5	1	3	2
3	5	1	3	1
5	5	1	1	1
	1	1	1	1

$\therefore$ LCM = 2 × 2 × 3 × 5 = 60

$$\frac{4}{5} = \frac{4 \times 12}{5 \times 12} = \frac{48}{60}, \frac{-1}{2} = \frac{-1 \times 30}{2 \times 30} = \frac{-30}{60}.$$

$$\frac{3}{6} = \frac{3 \times 10}{6 \times 10} = \frac{30}{60}, \frac{3}{4} = \frac{3 \times 15}{4 \times 15} = \frac{45}{60}.$$

Now, We have $\frac{48}{60}, \frac{-30}{60}, \frac{30}{60}$ and $\frac{45}{60}$.

Comparing the numerators,

$$\frac{-30}{60} < \frac{30}{60} < \frac{45}{60} < \frac{48}{60}$$

$$\frac{-1}{2} < \frac{3}{6} < \frac{3}{4} < \frac{4}{5}$$

Thus, the given rational numbers in ascending order are : $\frac{-1}{2}, \frac{3}{6}, \frac{3}{4}, \frac{4}{5}$.

(b) We have $\frac{-2}{7}, \frac{1}{6}, \frac{5}{8}$ and $\frac{3}{-5}$.

LCM of the denominators 7, 6, 8 and 5 is 840.
Converting the given rational numbers into equivalent rational numbers with denominator 840, we get :

2	7	6	8	5
2	7	3	4	5
2	7	3	2	5
3	7	3	1	5
5	7	1	1	5
7	7	1	1	1
	1	1	1	1

$\therefore$ LCM = 2 × 2 × 2 × 3 × 5 × 7
= 840

$$\frac{-2}{7} = \frac{-2 \times 120}{7 \times 120} = \frac{-240}{840}, \frac{1}{6} = \frac{1 \times 140}{6 \times 140} = \frac{140}{840},$$

$$\frac{5}{8} = \frac{5 \times 105}{8 \times 105} = \frac{525}{840}.$$

$$\frac{3}{-5} = \frac{3 \times 168}{-5 \times 168} = \frac{504}{-840} = \frac{-504}{840}.$$

Now, We have $\frac{-240}{840}, \frac{140}{840}, \frac{525}{840}, \frac{-504}{840}$

$$\frac{-504}{840} < \frac{-240}{840} < \frac{140}{840} < \frac{525}{840}$$

$$\frac{3}{-5} < \frac{-2}{7} < \frac{1}{6} < \frac{5}{8}$$

Thus, the given rational numbers in ascending order are : $\frac{3}{-5}, \frac{-2}{7}, \frac{1}{6}$ and $\frac{5}{8}$.

4. (a) –4 is smaller than $\frac{1}{2}$ because a negative number is smaller than any positive number.

(b) We have $\frac{-6}{11}$ and $\frac{6}{-12}$.

By cross multiplication,

$\frac{-6}{11} \times \frac{6}{-12}$

$-6 \times (-12)$ $\quad$ 6×11

72 $\quad$ 66

As 66 is smaller than 72, so $\frac{6}{-12}$ is smaller.

(c) $\frac{-2}{3}$ is smaller than $\frac{3}{5}$ because a negative number is smaller than any positive number.

(d) Similar work to be done as (c) above.

5. Two rational numbers between –5 and –1 are : –3 and –2.

6. There are infinite number of rational numbers between –1 and 1.

7. We have $\frac{-2}{3}$ and $\frac{1}{4}$.

LCM of the denominators 3 and 4 = 12.

$\therefore \frac{-2}{3} = \frac{-2 \times 4}{3 \times 4} = \frac{-8}{12}$ and $\frac{1}{4} = \frac{1 \times 3}{4 \times 3} = \frac{3}{12}$.

Now, we have $\frac{-8}{12}$ and $\frac{3}{6}$.

$\therefore$ Six rational numbers are : $\frac{-7}{12}, \frac{-6}{12}, \frac{-5}{12}, \frac{-4}{12}, \frac{-3}{12}$ and $\frac{-2}{12}$.

8. Given $x = \frac{-2}{5}$

$\therefore |x| = \left|\frac{-2}{5}\right| = \frac{2}{5}$

There are three rational numbers between $\frac{-2}{5}$ and $\frac{2}{5}$, i.e., $\frac{-1}{5}$, 0 and $\frac{1}{5}$.

To get four rational number, we make the denominator as 10.

$\frac{-2}{5} = \frac{-2 \times 2}{5 \times 2} = \frac{-4}{10}$ and $\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10}$.

Thus, the four required rational numbers are : $\frac{-3}{10}, \frac{-2}{10}, \frac{-1}{10}$ and $\frac{1}{10}$.

9. 3.5 and 3.9.

$\therefore 3.5 = \frac{35}{10}$ and $3.9 = \frac{39}{10}$

Three rational numbers between $\frac{35}{10}$ and $\frac{39}{10}$ are: $\frac{36}{10}, \frac{37}{10}$ and $\frac{38}{10}$.

10. We know that every number in the form $\frac{p}{q}$, where q ↑ 0 is a rational number

We can write zero (0) as $\frac{0}{5}, \frac{0}{17}, \frac{0}{33}, ...$.

Hence, zero is rational number.

Exercise 1.5

1. Other rational number = Product ÷ One rational number

$$= \frac{-56}{135} \div \frac{28}{-45} = \frac{\cancel{-56}^{-2}}{\cancel{135}_3} \times \frac{\cancel{-45}^{-1}}{\cancel{28}_1} = \frac{2}{3}.$$

2. Cost of 1 litre of petrol = ₹ $78\frac{3}{4}$

$\therefore$ cost of $5\frac{1}{2}$ litres of petrol = ₹ $\left(78\frac{3}{4} \times 5\frac{1}{2}\right)$

= ₹ $\left(\frac{315}{4} \times \frac{11}{2}\right)$ = ₹ $\left(\frac{315 \times 11}{4 \times 2}\right)$ = ₹ $\frac{3495}{8}$ = ₹ $433\frac{1}{8}$.

3. Sum of $\frac{-3}{14}$ and $\frac{2}{5} = \frac{-3 \times 5 + 2 \times 14}{70} = \frac{-15 + 28}{70} = \frac{13}{70}$. [LCM of 14 and 5 = 70]

Their difference = $\frac{2}{5} - \frac{-3}{14} = \frac{2 \times 14 + 3 \times 5}{70} = \frac{28 + 15}{70} = \frac{43}{70}$.

Dividing the difference by the sum, we get

$\frac{43}{70}$ √ $\frac{13}{70} = \frac{43}{\cancel{70}} \times \frac{\cancel{70}}{13} = \frac{43}{13}$.

Thus, the answer is $\frac{43}{13}$.

4. Sum of $\frac{13}{42}$ and $\frac{-5}{6} = \frac{13}{42} + \frac{-5}{6} = \frac{13 - 5 \times 7}{42} = \frac{13 - 35}{42} = \frac{-22}{42} = \frac{-1}{2}$.

Subtracting $\frac{-2}{9}$ from the sum, we get

$\frac{-11}{21} - \frac{-2}{9} = \frac{-11}{21} + \frac{2}{9} = \frac{-11 \times 3 + 2 \times 7}{63} = \frac{-33 + 14}{63} = \frac{-19}{63}$.

Thus, the answer is = $\frac{-19}{63}$.

5. Shubhi spends of her income on food = $\frac{1}{3}$

She spends of her income on rent = $\frac{2}{5}$

Part of her income she spends = $\frac{1}{3} + \frac{2}{5} = \frac{5 + 6}{15} = \frac{11}{15}$.

$\therefore$ Part of her income she saves = $1 - \frac{11}{15} = \frac{15-11}{15} = \frac{4}{15}$.

Thus, Shubhi saves $\frac{4}{15}$ of her income.

6. Given : Length of a rectangle = $50\frac{3}{4} = \frac{203}{4}$ m and its breadth = $40\frac{1}{4} = \frac{161}{4}$ m.

$\therefore$ Perimeter of the rectangle = $2 \times \left(\frac{203}{4} + \frac{161}{4}\right) = \frac{364}{4} \times 2 = \frac{728}{4} = 182$ m.

Area of the rectangle = length × breadth

$= \frac{203}{4} \times \frac{161}{4} = \frac{32683}{16} = 2042\frac{11}{16}\text{m}^2$.

Thus, the perimeter of the rectangle is 182m and its area is $2042\frac{11}{16}\text{m}^2$.

7. Distance travelled towards east = $8\frac{2}{5}$km = $\frac{42}{5}$km.

Distance travelled towards north = $4\frac{1}{2}$km = $\frac{9}{2}$km.

Distance travelled towards west = $4\frac{5}{8}$km = $\frac{37}{8}$km.

$\therefore$ Total distance travelled by Imran = $\left(\frac{42}{5} + \frac{9}{2} + \frac{37}{8}\right)$km

$= \frac{42 \times 8 + 9 \times 20 + 37 \times 5}{40}\text{km} = \frac{336 + 180 + 185}{40} = \frac{701}{40} = 17\frac{21}{40}$km.

8. Quantity of apples in the basket = $10\frac{1}{5}$kg = $\frac{51}{5}$kg.

Quantity of oranges in the basket = $8\frac{1}{9}$kg = $\frac{73}{9}$kg.

Total weight of the basket = $25\frac{4}{5}$kg = $\frac{129}{5}$kg.

Weight of quava in the basket $= \frac{129}{5} - \left(\frac{51}{5} + \frac{73}{9}\right)\text{kg} = \frac{129}{5} - \left(\frac{51 \times 9}{5 \times 9} + \frac{73 \times 5}{9 \times 5}\right)\text{kg}$

$= \frac{129}{5} - \left(\frac{459}{45} + \frac{365}{45}\right)\text{kg} = \frac{129}{5} - \left(\frac{459 + 365}{45}\right)\text{kg}$

$= \frac{129}{5} - \frac{824}{45}\text{kg} = \frac{1161 - 824}{45} = \frac{337}{45} = 7\frac{22}{45}$kg.

Thus, the weight of the guavas in the basket is $7\frac{22}{45}$kg.

9. Cost of 1 litre of petrol = ₹ $88\frac{3}{4}$ = ₹ $\frac{355}{4}$

Cost of $10\frac{1}{8}$ litres of petrol = ₹ $\frac{355}{4} \times 10\frac{1}{8}$ = ₹ $\frac{355}{4} \times \frac{81}{8}$ = ₹ $\frac{28755}{32}$ = ₹ $898\frac{19}{32}$.

Thus, Guneet spent ₹ $898\frac{19}{32}$.

10. Money the carpenter spent on snakes = ₹ $20\frac{2}{5}$ = ₹ $\frac{102}{5}$

Money he spent on food = ₹ $50\frac{1}{5}$ = ₹ $\frac{251}{5}$

Money he spent of repairing tools = ₹ $10\frac{1}{2}$ = ₹ $\frac{21}{2}$

Total money he spent = ₹ $\left(\frac{102}{5} + \frac{251}{5} + \frac{21}{2}\right)$ = ₹ $\left(\frac{353}{5} + \frac{21}{2}\right)$

= ₹ $\frac{353 \times 2 + 21 \times 5}{10}$ = ₹ $\frac{706 + 105}{10}$ = ₹ $\frac{811}{10}$ = ₹ $81\frac{1}{10}$.

Total earning of the carpenter = ₹650

Total money he spent = ₹ $81\frac{1}{10}$ = ₹ 81.10

Money he saved = ₹ (650 – 81.10) = ₹ 568.90

Thus, the carpenter saved = ₹ 568.90 = ₹ $568\frac{9}{10}$.

Revision Exercise

1. Similar work to be done as Q. 1 of Exercise 1.3.
2. Similar work to be done as Q. 2 of Exercise 1.3.
3. Similar work to be done as Q. 6 of Exercise 1.1.
4. We have $\left(\frac{-3}{8} \times \frac{2}{-3}\right) + \left(\frac{-3}{8} \times \frac{-4}{5}\right) = \frac{-3}{8} + \left(\frac{2}{-3} + \frac{-4}{5}\right)$

LHS = $\left(\frac{\not{-3}}{\not{8}_4} \times \frac{\not{2}^1}{\not{-3}}\right) + \left(\frac{-3}{\not{8}_2} \times \frac{\not{-4}^{-1}}{5}\right) = \frac{1}{4} + \frac{3}{10} = \frac{5+6}{20} = \frac{11}{20}$.

RHS = $\frac{-3}{8}\left(\frac{2}{-3} + \frac{-4}{5}\right) = \frac{-3}{8}\left(\frac{-2 \times 5 + -4 \times 3}{15}\right)$ [LCM of 3 and 5 = 15]

$= \frac{-3}{8}\left(\frac{-10 - 12}{15}\right) = \frac{\not{-3}^{-1}}{\not{8}_4} \times \frac{\not{-22}^{-11}}{\not{15}_5} = \frac{11}{20}$.

As LHS = RHS
Hence, verified.

5. Similar work to be done as Q.5. of Exercise 1.2.

6. (a) Multiplicative identity property.
   (b) Distributive property of multiplication over addition
   (c) Commutative property of multiplication
   (d) Associative property of addition.

7. (a) Given : $3\frac{1}{2} \times -2\frac{2}{3} = \frac{-8}{3} \times x$

$\therefore \frac{7}{2} \times \frac{-8}{3} = \frac{-8}{3} \times x$

$= \frac{7}{2} \times \frac{-8}{3} = \frac{-8}{3} \times \frac{7}{2}$ [Using commutative property]

Thus, the value of x is $\frac{7}{2}$.

   (b) Given $\frac{5}{9} + x = 0$

$\therefore \frac{5}{9} + \frac{-5}{9} = 0$ [Using additive inverse property]

Thus, the value of x is $\frac{-5}{9}$.

   (c) given $x \times \frac{7}{11} = 1$

$\therefore \frac{11}{7} \times \frac{7}{11} = 1$ [Using multiplicative property]

Thus, the value of x is $\frac{3}{5}$ $\frac{2}{3}$

8. (a) We have $\frac{3}{5}$ and $\frac{2}{3}$.

By cross multiplication,

$\frac{3}{5}$ $\frac{2}{3}$

3 × 3   2 × 5
9   10

As 10 is greater than 9, so $\frac{2}{3} > \frac{3}{5}$.

   (b) We have $\frac{-4}{7}$ and $\frac{-5}{9}$.

By cross multiplication,

$\frac{-4}{7}$ $\frac{-5}{9}$

–4 × 9   –5 × 7
–36   –35

As –35 is greater than –36, so $\frac{-5}{9}$ is greater.

(c) We have $\frac{6}{-11}$ and $\frac{3}{-2}$.

By cross multiplication,

$$\frac{6}{-11} \times \frac{3}{-2}$$

$6 \times (-2)$ $\quad$ $3 \times (-11)$

-12 $\quad$ -33

As –12 is greater than –33, so $\frac{6}{-11}$ is greater.

9. $-\left(2\frac{1}{5}\right) = -\frac{11}{5}$

Now $\frac{-11}{5} \times \frac{-5}{11} = 1$

As the product is 1, so $-\left(2\frac{1}{5}\right)$ is the multiplicative inverse of $\frac{-5}{11}$.

10. Similar work to be done as Q. 2 of Exercise 1.4.

11. (a Six rational numbers greater than 0 are : $\frac{1}{2}, \frac{4}{5}, \frac{6}{9}, \frac{5}{10}, \frac{4}{14}$ and $\frac{6}{19}$.

Note: As there are infinite number of rational numbers greater than 0, so the answer may be different.

(b) We have $\frac{-3}{4}$ and $\frac{1}{2}$.

LCM of the denominators 4 and 2 is 4.

$\therefore \frac{-3}{4} = \frac{-3 \times 1}{4 \times 1} = \frac{-3}{4}$ and $\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$.

Now, we $\frac{-3}{4}$ and $\frac{2}{4}$.

Thus, four rational numbers between $\frac{-3}{4}$ and $\frac{1}{2}$ are : $\frac{-2}{4}, \frac{-1}{4}, 0$ and $\frac{1}{4}$.

12. Given : Number of boys is class VII = 20

$\therefore$ Number of boys in class VIII = $2\frac{1}{2} \times 20 = \frac{5}{\cancel{2}} \times \cancel{20}^{10} = 5 \times 10 = 50$ boys.

Thus, there are 50 boys in class VIII.

Multiple Choice Question

1. The half of the sum of two rational number is always a rational number. Thus, the option (b) is correct.
2. Subtraction of rational numbers holds good for closure property. Thus, the option (c) is correct.
3. A rational numbers is not closed under division. Thus, the correct option is (d).

4. Multiplicative inverse of –8 = $\frac{1}{-8}$.

Additive inverse of –8 = 8

Product of these two results = $\frac{1}{-8} \times 8$ = –1. Thus, the correct option is (a).

5. Additive inverse of –6 is 6. Thus, the correct option is (c).

6. The absolute value of rational numbers is always positive. Thus the correct option is (d).

7. The division of rational numbers does not hold good for distributive property. Thus, the correct option is (d).

8. See the Answers given in the book.

9. See the Answers given in the book.

10. The product of a rational number and zero is always zero. Thus, the correct option is (a).

Mental Maths

A. See the **Answers** given in the book.

B. See the **Answers** given in the book.

Higher Order Thinking Skills (HOTS)

1. 1 is the only rational number which is equal to its multiplicative inverse.

2. Let the rational number be x.

Then its reciprocal = $\frac{1}{x}$

According to the question,

$\frac{1}{x} \times \frac{1}{5} = \frac{4}{6} \quad \Rightarrow x = \frac{20}{6} = \frac{10}{3}$.

Thus, the required rational number is $\frac{10}{3}$.

3. See the **Answers** given in the book.

2 Exponents and Powers

Exercise 2.1

1. (a) base = 3, exponent = 5

(b) base = –2, exponent = 2

(c) base = $\frac{-2}{7}$, exponent = 15

(d) base = $\frac{4}{11}$, exponent = –6

2. (a) $(4) \times (-4) \times (-4) \times (-4) \times (-4) = (-4)^5$

(b) $\frac{5}{-9} \times \frac{5}{-9} \times \frac{5}{-9} = \left(\frac{5}{-9}\right)^3$

(c) $\frac{125}{729} = \frac{5 \times 5 \times 5}{9 \times 9 \times 9} = \frac{5^3}{9^3} = \left(\frac{5}{9}\right)^3$

(d) We have $\frac{128}{2187}$.

By prime factorisation,

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$\therefore 128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$

3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

$\therefore 2187 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^7$

$\therefore \frac{128}{2187} = \frac{2^7}{3^7} = \left(\frac{2}{3}\right)^7$

3. (a) $(-5)^2 = (-5) \times (-5) = 25.$

(b) $6^4 = 6 \times 6 \times 6 \times 6 = 1296.$

(c) $\left(\frac{10}{11}\right)^3 = \frac{10^3}{11^3} = \frac{10 \times 10 \times 10}{11 \times 11 \times 11} = \frac{1000}{1331}$

(d) $3^2 \times 2^3 = 3 \times 3 \times 2 \times 2 \times 2 = 9 \times 8 = 72.$

4. Multiplicative inverse of :

(a) $\frac{1}{6^{-5}} = 6^5$

(b) $\frac{2}{3} = \frac{3}{2}$

(c) $\frac{-4}{7} = \frac{7}{-4}$

(d) $\left(\frac{2}{-9}\right)^2 = \left(\frac{-9}{2}\right)^2$

(e) $\left(\frac{1}{6}\right)^{-2} = 6^2$

5. (a) $5^{-3} = \frac{1}{5^3} = \frac{1}{5 \times 5 \times 5} = \frac{1}{125}.$

(b) $\left(\frac{4}{3}\right)^{-5} = \left(\frac{3}{4}\right)^5 = \frac{3^5}{4^5} = \frac{3 \times 3 \times 3 \times 3 \times 3}{4 \times 4 \times 4 \times 4 \times 4} = \frac{243}{1024}.$

(c) $\left(\frac{-2}{3}\right)^{-3} = \left(\frac{3}{-2}\right)^3 = \frac{3 \times 3 \times 3}{(-2) \times (-2) \times (-2)} = \frac{27}{-8}.$

(d) $\left(\frac{16}{25}\right)^{-2} = \left(\frac{25}{16}\right)^2 = \frac{25 \times 25}{16 \times 16} = \frac{625}{256}.$

(e) $6^{-2} \times 3^{-1} = \frac{1}{6^2} \times \frac{1}{3} = \frac{1}{6 \times 6} \times \frac{1}{3} = \frac{1}{6 \times 6 \times 3} = \frac{1}{108}.$

6. (a) $\left(\frac{3}{7}\right)^{-2} = \left(\frac{7}{3}\right)^2 = \frac{7^2}{3^2} = \frac{49}{9}.$

(b) $\frac{6}{5} \times \left(\frac{6}{5}\right)^{-1} = \frac{\cancel{6}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{6}} = 1.$

(c) $8 \times 2^{-4} = 8 \times \frac{1}{2^4} = 8 \times \frac{1}{2 \times 2 \times 2 \times 2} = \frac{\cancel{8}^{1}}{\cancel{16}_{2}} = \frac{1}{2}.$

(d) $25 \times 2^2 = 25 \times 4 = 100.$

(e) $3 \times (3 \times 3)^{-3} = 3\,(9)^{-3} = 3 \times \frac{1}{9^3} = \frac{\cancel{3}^{1}}{\cancel{9}_{3} \times 9 \times 9} = \frac{1}{243}.$

(f) $6 \times 6^{-1} = \cancel{6} \times \frac{1}{\cancel{6}} = 1.$

(g) $\left(\frac{3}{4}\right)^{-1} - \left(\frac{3}{4}\right)^{-1} = \frac{4}{3} - \frac{4}{3} = 0.$

(h) $(4^{-1} - 5^{-1})^{-1} = \left(\frac{1}{4} - \frac{1}{5}\right)^{-1} = \left(\frac{5-4}{20}\right)^{-1} = \left(\frac{1}{20}\right)^{-1} = 20.$ [LCM of 4 and 5 = 20]

Exercise 2.2

1. (a) $\left[\left(\frac{3}{5}\right)^{4}\right]^{-1} = \left(\frac{3}{5}\right)^{4 \times (-1)} = \left(\frac{3}{5}\right)^{-4} = \left(\frac{5}{3}\right)^{4}.$

(b) $\left[(-5)^{-4}\right]^{-3} = (-5)^{(-4) \times (-3)} = (-5)^{12}.$

(c) $3^7 \times 3^{-10} = 3^{7 + (-10)} = 3^{-3} = \frac{1}{3^3}.$

(d) $\left(\frac{4}{9}\right)^{-2} \div \left(\frac{16}{81}\right)^{3} = \left(\frac{4}{9}\right)^{-2} \div \left[\left(\frac{4}{9}\right)^{2}\right]^{3} = \left(\frac{4}{9}\right)^{-2} \div \left(\frac{4}{9}\right)^{2 \times 3}$

$= \left(\frac{4}{9}\right)^{-2} \div \left(\frac{4}{9}\right)^{6} = \left(\frac{4}{9}\right)^{-2-6} = \left(\frac{4}{9}\right)^{-8} = \left(\frac{9}{4}\right)^{8} = \left[\left(\frac{3}{2}\right)^{2}\right]^{8} = \left(\frac{3}{2}\right)^{2 \times 8} = \left(\frac{3}{2}\right)^{16}.$

2. (a) $\left(\frac{3}{8}\right)^{2} \times \left(\frac{4}{5}\right)^{-2} = \left(\frac{3}{8}\right)^{2} \times \left(\frac{5}{4}\right)^{2} = \frac{3}{8} \times \frac{3}{8} \times \frac{5}{4} \times \frac{5}{4} = \frac{225}{1024}.$

(b) $\left(\frac{6}{7}\right)^{2} \times \left(\frac{-2}{8}\right)^{-3} \times \left(\frac{2}{-7}\right)^{-3} = \left(\frac{6}{7}\right)^{2} \times \left(\frac{8}{-2}\right)^{3} \times \left(\frac{-7}{2}\right)^{3}$

$= \frac{6}{\cancel{7}} \times \frac{6}{\cancel{7}} \times \frac{\cancel{8}}{\cancel{-2}} \times \frac{8}{\cancel{-2}} \times \frac{\cancel{8}}{\cancel{-2}} \times \frac{\cancel{-7}^{-1}}{\cancel{2}} \times \frac{\cancel{-7}^{-1}}{\cancel{2}} \times \frac{-7}{\cancel{2}}$

$= \frac{36}{1} \times \frac{8}{-1} \times \frac{-7}{1} = 36 \times 8 \times 7 = 2016.$

(c) $\left(\frac{4}{5}\right)^{2} \div \left(\frac{2}{3}\right)^{-1} \times (-3)^{0} = \left(\frac{4}{5}\right)^{2} \div \left(\frac{3}{2}\right) \times 1 = \frac{4}{5} \times \frac{4}{5} \times \frac{2}{3} \times 1 = \frac{32}{75}.$

(d) $\left(\frac{6}{13}\right)^{0} \div \left(\frac{4}{9}\right)^{-2} = 1 \div \left(\frac{9}{4}\right)^{2} = \left(\frac{4}{9}\right)^{2} = \frac{16}{81}.$

3. (a) $6^0 \times 3^0 = 1$

(b) $\left[\left(\dfrac{-4}{5}\right)^0 + \left(\dfrac{1}{4}\right)^0\right] \times 5^0 = (1 + 1) \times 1 = 2.$

(c) $(4^0 - 3^0 + 5^0) \div (a^0 + b^0) = (1 - 1 + 1) \div (1 + 1) = 1 \div 2 = \dfrac{1}{2}.$

(d) $(3^2)^0 = (9)^0 = 1.$

(e) $(a^5)^0 = 0$

(f) $\left(\dfrac{1}{a}\right)^{-x} \times \left(\dfrac{1}{b}\right)^{-x} = a^x \times b^x = (ab)^x.$

4. (a) $\left(\dfrac{4}{5}\right)^{-3} \times \left(\dfrac{4}{5}\right)^{-4} = \left(\dfrac{4}{5}\right)^{x}$

$\Rightarrow \left(\dfrac{4}{5}\right)^{-3 + (-4)} = \left(\dfrac{4}{5}\right)^{x}$

$\Rightarrow \left(\dfrac{4}{5}\right)^{-3 - 4} = \left(\dfrac{4}{5}\right)^{x}$

$\Rightarrow \left(\dfrac{4}{5}\right)^{-7} = \left(\dfrac{4}{5}\right)^{x}$

$\Rightarrow -7 = x$ [Bases are same.]

Thus, the value of x is –7.

(b) $\left(\dfrac{9}{14}\right)^{3} \div \left(\dfrac{9}{14}\right)^{5} = \left(\dfrac{9}{14}\right)^{x\quad 4}$

$\Rightarrow \left(\dfrac{9}{14}\right)^{3\quad 5} = \left(\dfrac{9}{14}\right)^{x\quad 4}$

$\Rightarrow \left(\dfrac{9}{14}\right)^{2} = \left(\dfrac{9}{14}\right)^{x\quad 4}$

$\Rightarrow -2 = x - 4$ [Bases are same.]

$\Rightarrow -2 + 4 = x$

$\Rightarrow 2 = x$

Thus, the value of x is 2.

(c) $2^{2x-4} = (16)^{x-2}$

$\Rightarrow 2^{2x-4} = (2^4)^{x-2}$

$\Rightarrow 2^{2x-4} = 2^{4x-8}$

$\Rightarrow 2^{x-4} = 4^{x-8}$ [Bases are same.]

$\Rightarrow 4 + 8 = 4x - 2x$

$\Rightarrow 4 = 2x = x = 4 \div 2 = 2$

Thus, the value of x is 2.

(d) $x^7 \div x^5 = \dfrac{36}{49}$

$$\Rightarrow x^{7-5} = \frac{36}{49}$$

$$\Rightarrow x^2 = \frac{36}{49} \qquad \Rightarrow x^2 = \left(\frac{6}{7}\right)^2 \qquad \Rightarrow x^2 = \frac{6}{7}$$ [Powers are same.]

Thus, the value of x is $\frac{6}{7}$.

5. To get the required number, we will divide 3 by $\left(\frac{3}{7}\right)^{-2}$

$\therefore$ Required number $= 3 \div \left(\frac{3}{7}\right)^{-2} = 3 \div \left(\frac{7}{3}\right)^2 = 3 \div \frac{49}{9} = 3 \times \frac{9}{49} = \frac{27}{49}$.

Thus, the required number is $\frac{27}{49}$.

6. Let the required number be x.

Then $(-5)^3 \div x = 25^{-2}$

$x = \frac{(-5)^3}{25^{-2}} = (-5)^3 \times 25^2 = (-5)^3 \times (-5^2)^2 = (-5)^3 \times (-5)^4 = -5^7$.

7. $\frac{p}{q} = \left(\frac{4}{9}\right)^4 \div \left(\frac{4}{9}\right)^3 = \left(\frac{4}{9}\right)^{4-3} = \left(\frac{4}{9}\right)^1 = \frac{4}{9}$ ...(i)

$\therefore \left(\frac{p}{q}\right)^{-2} = \left(\frac{4}{9}\right)^{-2} = \left(\frac{9}{4}\right)^2 = \frac{81}{16}$. [From (i)]

Thus, the value of $\left(\frac{9}{4}\right)^2$ is $\frac{81}{16}$.

8. (a) $\left[\left(\frac{7}{8}\right)^{-3} \div \left(\frac{7}{8}\right)^{-3}\right] \div \left(\frac{3}{8}\right)^{-3} = \left(\frac{7}{8}\right)^{-3-(-3)} \div \left(\frac{8}{3}\right)^{-3}$

$$= \left(\frac{7}{8}\right)^{-3+3} \div \left(\frac{8}{3}\right)^3 = \left(\frac{7}{8}\right)^0 \div \left(\frac{8}{3}\right)^3 = 1 \div \left(\frac{8}{3}\right)^3 = 1 \times \left(\frac{3}{8}\right)^3 = \frac{27}{512}.$$

(b) $\left[\left(\frac{5}{2}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right]^{-1} = \left(\frac{2}{5} - \frac{4}{1}\right)^{-1} = \left(\frac{2-20}{5}\right)^{-1} = \left(\frac{-18}{5}\right)^{-1} = \frac{-5}{18}$.

(c) $\left(8^{-1} \times 4^{-1}\right)^{-1} = \left(\frac{1}{8} \times \frac{1}{4}\right)^{-1} = \left(\frac{1}{32}\right)^{-1} = 32$.

(d) $\left[\left\{\left(\frac{-1}{4}\right)^2\right\}^{-2}\right]^{-1} = \left[\left\{\frac{-1}{4}\right\}^{-4}\right]^{-1} = \left[\frac{-1}{4}\right]^{(-4)\times(-1)} = \left[\frac{-1}{4}\right]^4 = \frac{1}{256}$.

9. $\left[\left(\frac{3}{4}\right)^2\right]^3 \times \left(\frac{1}{4}\right)^{-2} \times 2^{-1} \times \left(\frac{1}{6}\right)^{-1} = \left(\frac{3}{4}\right)^{2\times3} \times 4^2 \times \frac{1}{\cancel{2}_1} \times \cancel{6}^3$

$$= \left(\frac{3}{4}\right)^6 \times 16 \times 3 = \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{\cancel{4}} \times \frac{3}{\cancel{4}} \times \cancel{16} \times 3 = \frac{2187}{256}.$$

10. $\left(\frac{6}{11}\times\frac{-22}{21}\right)^{-4}=\left(\frac{6}{11}\times\frac{21}{-22}\right)^{4}$

LHS $=\left(\frac{\cancel{6}^{2}}{\cancel{11}_{1}}\times\frac{\cancel{-22}^{2}}{\cancel{21}_{7}}\right)^{-4}=\left(\frac{2\times(-2)}{7}\right)^{-4}=\left(\frac{-4}{7}\right)^{-4}=\left(\frac{7}{-4}\right)^{4}$

RHS $=\left(\frac{\cancel{11}^{1}}{\cancel{6}_{2}}\times\frac{\cancel{21}^{7}}{\cancel{-22}_{-2}}\right)^{4}=\left(\frac{7}{-4}\right)^{4}$

$\therefore$ LHS = RHS

Hence, verified.

Exercise 2.3

1. In standard form :
   (a) $75.39 = 7.539 \times 10^1$
   (b) $4800000 = 4.8 \times 10^6$
   (c) $36500 = 3.65 \times 10^4$
   (d) $962 \times 10^5 = 9.6^2 \times 10^2 \times 10^5 = 9.62 \times 10^7$
   (e) $0.000082 = 8.2 \times 10^{-5}$
   (f) $0.00000005 = 5.0 \times 10^{-8}$
   (g) $0.0000005976 = 5.976 \times 10^{-7}$
   (h) $0.0869725 = 8.69725 \times 10^{-2}$

2. In the usual form :
   (a) $4.286 \times 10^3 = 4286$
   (b) $9.4872 \times 10^8 = 948720000$
   (c) $5.6 \times 10^5 = 560000$
   (d) $8.364 \times 10^4 = 83640$
   (e) $6.38 \times 10^{-4} = 0.000638$
   (f) $9 \times 10^{-6} = 0.000009$
   (g) $1.7 \times 10^{-5} = 0.000017$
   (h) $6245 \times 10^{10} = 62450000000000$

3. (a) Volume of water on the earth's surface = 1353000000 = 1.353×10^9 km^3.
   (b) 1 Angstrom = $\frac{1}{10000000000}$ m = 1.0×10^{-10} m.
   (c) Thickness of an iron plate = 0.003 mm = 3.0×10^{-3} mm
   (d) Length of a cell = 0.0000000065 mm = 6.5×10^{-9} mm.
   (e) Distance between Delhi and Howrah = 15286000 m = 1.5286×10^7 m.

Revision Exercise

1 (a) $(-6)^{-2}=\frac{1}{(-6)^2}$ (b) $3^{-5}=\frac{1}{3^5}$

(c) $\left(\frac{1}{5}\right)^{-6}=5^6$ (d) $\left(\frac{12}{7}\right)^{-3}=\left(\frac{7}{12}\right)^{3}$

2. (a) $\frac{25}{36} = \frac{5^2}{6^2} = \left(\frac{5}{6}\right)^2 = \left(\frac{6}{5}\right)^{-2}$

(b) We have 6561.

By prime factorisation,

3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

$\therefore$ 656 = $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^8 = \left(\frac{1}{3}\right)^{-8}$.

(c) We have $\frac{-216}{125}$.

By prime factorisation,

2	216
2	108
2	54
3	27
3	9
3	3
	1

5	125
5	25
5	5
	1

$\therefore 125 = 5 \times 5 \times 5 = 5^3$

$\therefore 216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3\, 3^3 = 6^3$

$\therefore -216 = -6^3$

$\therefore \frac{-216}{125} = \frac{(-6)^3}{5^3} = \left(\frac{-6}{5}\right)^3 = \left(\frac{-5}{6}\right)^{-3}$

(d) We have $\frac{100000}{161051} = \frac{10^5}{11^5} = \left(\frac{10}{11}\right)^5 = \left(\frac{11}{10}\right)^{-5}$.

3. (a) $25^8 \div 25^3 = 25^{8-3} = 25^5 = (5^2)^5 = 5^{2 \times 5} = 5^{10}$.

(b) $\left(\frac{-3}{8}\right)^4 \div \left(\frac{-3}{8}\right)^{-2} = \left(\frac{-3}{8}\right)^{4-(-2)} = \left(\frac{-3}{8}\right)^{4+2} = \left(\frac{-3}{8}\right)^6$.

(c) $\frac{-5}{10} \div \left(\frac{-5}{10}\right)^3 = \left(\frac{-5}{10}\right)^{1-3} = \left(\frac{-5}{10}\right)^{-2} = \left(\frac{-1}{2}\right)^{-2}$.

(d) $(-15)^{-10} \div (-15)^8 = (-15)^{-10-8} = (-15)^{-18}$.

4. (a) $6^{-9} \div 6^{-2} = 6^{-9+2} = 6^{-7} = \frac{1}{6^7}$.

(b) $8^{-1} \times 8 = 8^{-1+1} = 8^0 = 1$

(c) $\left(\frac{-2}{11}\right)^{-2} \times \left(\frac{-2}{11}\right)^{-2} \times \left(\frac{-2}{11}\right)^{-6} = \left(\frac{-2}{11}\right)^{-2-2-6} = \left(\frac{-2}{11}\right)^{-10} = \left(\frac{-11}{2}\right)^{10}$.

(d) $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \left(\frac{1}{2}\right)^{1+1+1} = \left(\frac{1}{2}\right)^{3}$.

5. (a) $(3^2)^{-2} = 3^{2\times(-2)} = 3^{-4} = \frac{1}{3^4} = \frac{1}{81}$.

(b) $\left(8^6\right)^{\frac{1}{3}} = 8^{6\times\frac{1}{3}} = 8^2 = 64$.

(c) $\left[\left(\frac{12}{20}\right)^2\right]^{-1} = \left[\left(\frac{3}{5}\right)^2\right]^{-1} = \left(\frac{3}{5}\right)^{2\times(-1)} = \left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^{2} = \frac{25}{9}$.

(d) $\left[\left(\frac{-41}{28}\right)^0\right]^{-5} = \left(\frac{-41}{28}\right)^{0\times(-1)} = \left(\frac{-41}{28}\right)^{0} = 1$.

6 (a) $3 \times \frac{1}{15} \times \frac{1}{3^{-1}} = \cancel{3}^1 \times \frac{1}{\cancel{15}_5} \times 3 = \frac{3}{5}$.

(b) $(4 \times 2^{-2} \times 5^0) \div 6^0 = \left(4 \times \frac{1}{2^2} \times 1\right) \div 1 = \left(\cancel{4} \times \frac{1}{\cancel{4}}\right) \div 1 = 1 \div 1 = 1$.

(c) $\left[\left(1^2 + 2^2 + 3^2\right)^{-1}\right]^0 = \left[\left(1 + 4 + 9\right)^{-1}\right]^0 = \left(14^{-1}\right)^0 = \left(\frac{1}{14}\right)^0 = 1$.

(d) $3^2 \times 3^{-2} \div 3^2 = 3^2 \times 3^{-2-2} = 3^2 \times 3^{-4} = 3^{2+(-4)} = 3^{2-4} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$.

7. (a) $x^2 \times y^2 = (xy)^2$

(b) $\left(\frac{2}{5}\right)^2 \times \left(\frac{2}{5}\right)^4 = \left(\frac{2}{5}\right)^{2\ \ 4} = \left(\frac{2}{5}\right)^{-2} = \left(\frac{5}{2}\right)^{2} = \frac{25}{4}$.

(c) $\left(\frac{6}{7}\right)^{-3} \times \left(\frac{12}{14}\right)^4 = \left(\frac{6}{7}\right)^{-3} \times \left(\frac{6}{7}\right)^4 = \left(\frac{6}{7}\right)^{-3+4} = \left(\frac{6}{7}\right)^{1} = \frac{6}{7}$.

(d) $(4^{-2} \times 3^2)^{-2} = \left(\frac{1}{4^2} \times 3^2\right)^{-2} = \left(\frac{1}{16} \times 9\right)^{-2} = \left(\frac{9}{16}\right)^{-2} = \left(\frac{16}{9}\right)^{2} = \frac{256}{81}$.

(e) $\left(5^{-2} \times 25\right)^{-1} + \frac{3}{5} = \left(\frac{1}{5^2} \times 25\right)^{-1} + \frac{3}{5} = \left(\frac{1}{\cancel{25}} \times \cancel{25}\right)^{-1} + \frac{3}{5} = (1)^{-1} + \frac{3}{5} = \frac{1}{1} + \frac{3}{5} = \frac{8}{5}$.

(f) $\left[\left(\frac{9}{5}\right)^2 \times \left(\frac{125}{81}\right)^4\right] \div \left(\frac{5}{9}\right)^2 = \left[\frac{\cancel{81}}{\cancel{25}_1} \times \frac{\cancel{125}^5}{\cancel{81}} \times \frac{125}{81}\right] \div \frac{25}{81} = \frac{5 \times \cancel{125}^5}{\cancel{81}} \times \frac{\cancel{81}}{\cancel{25}_1} = 5 \times 5 = 25$.

(g) $[2^5 \div 2^{-5}] \times 2^{-5} = [2^{5+5}] \times 2^{-5} = 2^{10-5} = 2^5 = 32$.

(h) $[3^{-7} \div 3^{-4}] \div 6^{-5} = 3^{-7+4} \div 6^{-5} = 3^{-3} \div \frac{1}{65} = \frac{1}{3^3} \times 6^5 = \frac{7776}{27} = 288$.

(i) $(-4)^{-3} \times (-4)^{-3} \div (-4)^{-3} = (-4)^{-3-3+3} = (-4)^{-3} = \frac{1}{-4^3} = \frac{1}{-64}$.

8. (a) $\left(\frac{5}{8}\right)^{2} \times \left(\frac{5}{8}\right)^{5} = \left(\frac{5}{8}\right)^{2x \quad 1} = \left(\frac{5}{8}\right)^{2 \quad 5} = \left(\frac{5}{8}\right)^{2x \quad 1} = \left(\frac{5}{8}\right)^{3} = \left(\frac{5}{8}\right)^{2x \quad 1}$

As bases are same, so their exponents will be equal.

$\therefore -3 = 2x - 1$

$\Rightarrow -2x = -1 + 3 = 2$

$\Rightarrow x = -1.$

Thus, the value of x is –1.

(b) $x^6 \div x^2 = \frac{16}{81}$

$\Rightarrow x^{6-2} = \frac{16}{81}$

$\Rightarrow x^4 = \frac{2^4}{3^4}$

$\Rightarrow x = \frac{2}{3}.$ [∵ Powers are same.]

Thus, the value of x is $\frac{2}{3}$.

9. The standard form of :

(a) 37492.8 is 3.74928×10^4.

(b) 34900000 is 3.49×10^7.

(c) 0.00000482 is 4.82×10^{-6}.

10. Let the required number be x.

Then $(-4)^{-1} \times x = (8)^{-1}$

$\Rightarrow \frac{x}{-4} = \frac{1}{8} \quad \Rightarrow x = \frac{-4}{8} = \frac{-1}{2}.$

Thus, the required number to be multiplied is $\frac{-1}{2}$.

Multiple Choice Questions

1. $\frac{-1}{3^{-2}} = -1 \times 3^2 = -1 \times 9 = -9$. Thus, the correct option is (b).

2. $(-4)^{-3} = \frac{1}{(-4)^3} = \frac{1}{-64}$. Thus, the correct option is (d).

3. $\left[(-4)^{-2}\right]^{-3} = (-4)^{(-2)\times(-3)} = (-4)^6$. Thus, the correct option is (b).

4. The standard form of 0.00000043 = 4.3×10^{-7}. Thus, the correct option is (a).

5. $4^4 \times (16)^{-1} = 4^4 \times (4^2)^{-1} = 4^4 \times 4^{-2} = 4^{4-2} = 4^2 = 16$.

Thus, the correct option is (b).

6. The reciprocal of $\left(\frac{3}{7}\right)^{-4} = \left(\frac{3}{7}\right)^{4}$. Thus, the correct option is (b).

7. $\left(1^{-3} + 2^{-3} + 3^{-3}\right)^0 = 1$. Thus, the correct option is (c).

8. $\frac{1}{3^{-2}} = 3^2 = 9$. Thus, the correct option is (b).

9. $\left(\frac{3}{8}\right)^{-7} \div \left(\frac{3}{8}\right)^{-2} = \left(\frac{3}{8}\right)^{-7+2} = \left(\frac{3}{8}\right)^{-5}$. Thus, the correct option is (c).

10. $(2^{-1} + 6^{-1}) \div 4^0 = \left(\frac{1}{2} + \frac{1}{6}\right) \div 1 = \frac{3+1}{6} = \frac{4}{6} = \frac{2}{3}$. Thus, the correct option is $\frac{2}{3}$.

Mental Maths

A. See the **Answers** given in the book.

B. 1. $\frac{1}{125} = \frac{1}{5 \times 5 \times 5} = \frac{1}{5^3} = 5^{-3}$.

2. $\left[\left(\frac{3}{7}\right)^{-4}\right] = 1$.

3. $x^a \times x^a = x^{a+a} = x^{2a}$.

4. $25^{-3} = (5^2)^{-3} = 5^{-6}$.

5. $x^a - x^a = 0$

Thus, we get 1. (iii), 2. (v), 3. (iv), 4. (ii), 5. (i)

Higher Order Thinking Skills (HOTS)

1. $$\frac{2^{m+2} \times 3^{2m-n} \times 3^{n+1} \times 2^m}{3^{1+m} \times 2^{2m+1} + 3^m} = \frac{2^{m+2+m} \times 3^{2m-n+n+1}}{3^{1+m+m} \times 2^{2m+1}} = \frac{2^{2m+2} \times 3^{2m+1}}{3^{2m+1} \times 2^{2m+1}}$$

$= 2^{2m+2-2m-1} \times 3^{2m+1-2m-1} = 2^1 \times 3^0 = 2 \times 1 = 2.$

2. $$\left(\frac{x^a}{x^b}\right)^2 \times \left(\frac{xc}{x^b}\right)^{-2} \times \left(\frac{x^c}{x^a}\right)^2 = \frac{x^{2a}}{x^{2b}} \times \frac{x^{2b}}{x^{2c}} \times \frac{x^{2c}}{x^{2a}} = 1.$$

3 Squares and Square Roots

Exercise 3.1

1. The units digit of the square of :

(a) 52 is 4 (b) 36 is 6 (c) 532 is 4 (d) 9563 is 9

2. (a) The square of 15 = 15 × 15 = 225.

(b) The square of 78 = 78 × 78 = 6084.

```
  78
 ×78
----
 624
5460
----
6084
----
```

∴ The square of 78 is 6084.

(c) The square of 99 = 99 × 99.

```
  99
 ×99
----
 891
8910
----
9801
----
```

∴ The square of 99 is 9801.

(d) Square of 108 = 108 × 108.

$$\begin{array}{r} 108 \\ \times 108 \\ \hline 864 \\ 0000 \\ 10800 \\ \hline 10864 \\ \hline \end{array}$$

Thus, the square of 108 is 10864.

3. We know that a number having 2, 3 7 or 8 at ones place can not be a perfect square. Thus :
   (a) 698 is not a perfect square.
   (b) 8100 is a perfect square.
   (c) 24964 is a perfect square.
   (d) 67500 is a perfect square.

4. We know that a number having n digits has either 2n or 2n – 1 digits. Thus, the possible number of digits is the square of :
   (a) 19 is 3 or 4. (b) 153 is 5 or 6. (c) 2031 is 7 or 8. (d) 13502 is 9 or 10.

5. We know that the square of an even number is even and that of an odd number is odd.
   (a) The square of 156 is an even number.
   (b) The square of 625 is an odd number.
   (c) The square of 2401 is an odd number.
   (d) The square of 1089 is an odd number.
   (e) The square of 1525 is an odd number.
   (f) The square of 7395 is an odd number.
   (g) The square of 10404 is an even number.
   (h) The square of 6561 is an odd number.

6. For every natural number m, we have the Pythagorean triplet as : $2m$, $m^2 - 1$, $m^2 + 1$, where $m > 1$.

   Note: Another Pythagorem triplet with 5 as one number is 3, 4, 5.

   (a) Let m be 5.
   Then $m^2 = 5 \times 5 = 25$
   $2m = 2 \times 5 = 10$
   $m^2 - 1 = 25 - 1 = 24$
   $m^2 + 1 = 25 + 1 = 26$
   Thus, the Pythagorean triplet is 10, 24, 26.

   (b) Let m be 10. Then $2m = 2 \times 10 = 20$.
   $m^2 = 10 \times 10 = 100$
   $m^2 - 1 = 100 - 1 = 99$ and $m^2 + 1 = 100 + 1 = 101$.
   Thus, the Pythagorean triplet is 20, 99, 101.
   Another Pythagorean triplet with 10 as one number is 6, 8, 10.

   (c) Similar work to be done. Also see the **Answers** given in the above.

   (d) Similar work to be done. Also see the **Answers** given in the above.

7. (a) We know that the sum of n odd numbers is equal to n^2. Here, n = 9.
   ∴ The sum of these odd numbers = $9^2 = 9 \times 9 = 81$.

   (b) Here, n = 7
   ∴ The sum of these odd numbers = $7^2 = 7 \times 7 = 49$.

8. We know that between the squares of two consecutive numbers n and n + 1, there exists 2n non-perfect squares numbers.

(a) We have 6^2 and 7^2. Here, n = 6

$\therefore$ Numbers lying between 6^2 and $7^2 = 2 \times 6 = 12$.

(b) We have 10^2 and 11^2. Here, n = 10

$\therefore$ Numbers lying between 10^2 and $11^2 = 2 \times 10 = 20$.

(c) Similar work to be done.

(d) Similar work to be done.

9. (a) $15^2 = 15 \times 15 = 225 = 3 \times 5 \times 15 = 3 \times 75$.

(b) $9^2 = 9 \times 9 = 81 = 3 \times 3 \times 9 = 3 \times 27$.

(c) $18^2 = 18 \times 18 = 3 \times 6 \times 18 = 3 \times 108$.

(d) $20^2 = 20 \times 20 = 4 \times 5 \times 20 = 4 \times 100$.

(e) $8^2 = 8 \times 8 = 4 \times 2 \times 8 = 4 \times 16$.

10. (a) $17^2 = 17 \times 17 = 289 = 144 + 145$.

(b) $35^2 = 35 \times 35 = 1225 = 612 + 613$.

(c) $11^2 = 11 \times 11 = 121 = 60 + 61$.

(d) $27^2 = 27 \times 27 = 729 = 364 + 365$.

(e) $19^2 = 19 \times 19 = 361 = 180 + 181$.

Exercise 3.2

1. (a) We have 423.

By prime factorisation,

3	423
3	141
47	47
	1

$\therefore 423 = 3 \times 3 \times 47$

Here, 47 is left unpaired.

Thus, 423 is not a perfect square.

(b) We have 650.

By prime factorisation,

2	650
5	325
5	65
13	13
	1

$\therefore 650 = 2 \times 5 \times 5 \times 13$

Here, 2 and 13 are unpaired.

Thus, 650 is not a perfect square.

(c) We have 729.

By prime factorisation,

3	729
3	243
3	81
3	27
3	9
3	3
	1

$\therefore 729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$

All the factors are grouped in pairs.

Thus, 729 is not a perfect square.

(d) We have 810.

By prime factorisation,

2	810
3	405
3	135
3	45
3	15
5	5
	1

$\therefore 810 = 2 \times 3 \times 3 \times 3 \times 3 \times 5$

Here, 2 and 5 are unpaired.

Thus, 810 is not a perfect square.

(e) We have 343.

By prime factorisation,

7	343
7	49
7	7
	1

$\therefore$ 343 = 7 × 7 × 7

Here, 7 is unpaired. Thus, 343 is not a perfect square.

2. (a) We have 100.

Subtracting successive odd numbers,

100 – 1 = 99
99 – 3 = 96
96 – 5 = 91
91 – 7 = 84
84 – 9 = 75
75 – 11 = 64
64 – 13 = 51
51 – 15 = 36
36 – 17 = 19
19 – 19 = 0

Here, subtraction takes 10 steps to reach 0. Thus, the square root of 100 is 10.

(b) We have 64.

Subtracting successive odd numbers,

64 – 1 = 63
63 – 3 = 60
60 – 5 = 55
55 – 7 = 48
48 – 9 = 39
39 – 11 = 28
28 – 13 = 15
15 – 15 = 0

Here, subtraction takes 8 steps to reach 0. Thus, the square root of 64 is 8.

(c) We have 225.

Subtracting successive odd numbers,

225 – 1 = 224
224 – 3 = 221
221 – 5 = 216
216 – 7 = 209
209 – 9 = 200
200 – 11 = 189
189 – 13 = 176
176 – 15 = 161
161 – 17 = 144
144 – 19 = 125

125 – 21 = 104
104 – 23 = 81
81 – 25 = 56
56 – 27 = 29
29 – 29 = 0

Here, subtraction takes place 15 times to reach 0. Thus, the square root of 225 is 15.

(d) We have 144.

Subtracting successive odd numbers,

144 – 1 = 143
143 – 3 = 140
140 – 5 = 135
135 – 7 = 128
128 – 9 = 119
119 – 11 = 108
108 – 13 = 95
95 – 15 = 80
80 – 17 = 63
63 – 19 = 44
44 – 21 = 23
23 – 23 = 0

Here, subtraction takes place 12 times to reach 0. Thus, the square root of 144 is 12.

(e) Similar work to be done. (f) Similar work to be done.

4. (a) We have 675.

By prime factorisation,

3	675
3	225
3	75
5	25
5	5
	1

$\therefore$ 675 = 3 × 3 × 3 × 5 × 5

Here, the prime factor 3 is left unpaired. Thus, the required number to be multiplied is 3.

(b) We have 1800.

By prime factorisation,

2	1800
2	900
2	450
3	225
3	75
5	25
5	5
	1

$\therefore$ 1800 = 2 × 2 × 2 × 3 × 3 × 5 × 5

Here, the prime factor 2 is left unpaired. Thus, the required number to be multiplied is 2.

(c) We have 686.

By prime factorisation,

2	686
7	343
7	49
7	7
	1

$\therefore$ 686 = 2 × 7 × 7 × 7

Here, the prime factor 2 and 7 are left unpaired. Thus, the required number to be multiplied is 2 × 7 = 14.

(d) Similar work to be done.

5. Number of soldiers in each row = $\sqrt{1156}$

$\sqrt{1156} = \sqrt{2 \times 2 \times 17 \times 17}$

= 2 × 17 = 34

2	1156
2	578
17	289
17	17
	1

$\therefore$ 1156 = 2 × 2 × 17 × 17

Thus, the number of soldiers in each row is 34.

6. LCM of the given numbers 6, 12, 15 and 18 = 2 × 2 × 3 × 3 × 5 = 180

2	6	12	15	18
2	3	6	15	9
3	3	3	15	9
3	1	1	5	3
5	1	1	5	1
	1	1	1	1

$\therefore$ LCM = 2 × 2 × 3 × 3 × 5 = 180

Here, the prime factor 5 is left unpaired. To, to make 180 a perfect square, we have to multiply it by 5.

$\therefore$ 180 × 5 = 900.

Thus, the required smallest square number is 900.

7. Area of the square = 1600 cm^2

$Side^2$ = 1600 cm^2

side = $\sqrt{1600}$ cm

= $\sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5}$ cm

= 2 × 2 × 2 × 5 = 40 cm

Thus, the dimension (side) of the square is 40 cm.

2	1600
2	800
2	400
2	200
2	100
2	50
5	25
5	5
	1

8. LCM of the given numbers 2, 3, 4 and 6 = 2 × 2 × 3 = 12

Here, the prime factor 3 is left unpaired.

So, to make 12 a perfect square, we have to multiply it by 3.

2	2	3	4	6
2	1	3	2	3
3	1	3	1	3
	1	1	1	1

$\therefore$ LCM = 2 × 2 × 3 = 12.

∴ 12 × 3 = 36.

Thus, the required smallest square number is 36.

9. (a) We have 1250.

By prime factorisation,

2	1250
5	625
5	125
5	25
5	5
	1

1250 = 2 × 5 × 5 × 5 × 5

The prime factor 2 is left unpaired.

Thus, the given number 1250 should be divided by 2 to make it a perfect square.

Now, 1250 ÷ 2 = 625, which is a perfect square.

625 = 5 × 5 × 5 × 5 = 5 × 5 = 25.

(b) We have 2048. By prime factorisation,

2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

2048 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2

The prime factor 2 is left unpaired.

Thus, the given number 2048 should be divided by 2 to make it a perfect square.

Now, 2048 ÷ 2 = 1024, which is a perfect square.

1024 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2

$\sqrt{1024}$ = 2 × 2 × 2 × 2 × 2 = 32.

(c) Similar work to be done.

(d) Similar work to be done.

Exercise 3.3

1. We know that a number having *n* digits has either 2n or 2n – 1 digits in its square.

(a) The number 48841 has five digit, so the number of digits in its square root is 3.

(b) 961 has three digits, so the number of digits in its square root must be 2.

(c) 567009 has six digits, so the number of digits in its square root must be 3.

(d) 649636 has six digits, so the number of digits in its square root must be 3.

(e) 23716 has five digits, so the number of digits in its square root must be 3.

2. (a)

	123
1	15129
	1
22	051
	44
243	0729
	729
	0

∴ $\sqrt{15129}$ = 123.

(b)

	1235
1	1525225
	1
22	52
	44
243	852
	729
2465	12325
	12325
	0

∴ $\sqrt{1525225}$ = 1235.

(c)

	95
9	9025
	81
185	925
	925
	0

∴ $\sqrt{9025}$ = 95.

(d)

	50
5	2500
	25
100	000
	000
	0

$\therefore \sqrt{2500} = 50.$

(e)

	250
2	62500
	4
45	225
	225
500	000
	000
	0

$\therefore \sqrt{62500} = 250.$

(f)

	59
5	3481
	25
109	981
	981
	0

$\therefore \sqrt{3481} = 59.$

(g)

	2301
2	5294601
	4
43	129
	129
4601	04601
	4601
	0

$\therefore \sqrt{5294601} = 2301.$

(h)

	101
1	10201
	1
201	00201
	201
	0

$\therefore \sqrt{10201} = 101.$

3. (a) We have 15140.

Let us find the square root of 15140.

	123
1	15140
	1
22	051
	44
243	740
	729
	11

We get the remainder 11.
Thus, the required number to be subtracted is 11.

(b) We have 40810.
Let us find the square root of 40841.

	202
2	40810
	4
402	00810
	804
	6

We get the remainder 6.
Thus, the required number to be subtracted is 6.

(c) We have 1175100.

Let us find the square root.

	1084
1	1175100
	1
208	01751
	1664
2164	8700
	8656
	44

We get 44 as the remainder.

Thus, the required number to be subtracted is 44.

(d) We have 90087.

Let us find the square root.

	300
3	90087
	9
600	00087
	0000
	87

We get 87 as the remainder.

Thus, the required number to be subtracted is 87.

4. (a) We have 4878.

Let us find the square root.

	69
6	4878
	36
129	1278
	1161
	117

Clearly, $69^2 < 4878 < 70^2$

Now $70^2 = 70 \times 70 = 4900$

$\therefore$ 4900 – 4878 = 22.

Thus, the required least number that must be added to 4878 to make it a perfect square is 22.

(b) We have 10000.

Let us find the square root.

	100
1	10000
200	0000
	0000
	0

As the remainder is 0, so the given number is already a perfect square.

(c) We have 4355.

Let us find the square root.

	65
6	4355
	36
125	755
	625
	130

Clearly, $65^2 < 4355 < 66^2$

Now $66^2 = 66 \times 66 = 4356$.

4356 – 4355 = 1

Thus, the least number to be added to 4355 to make it a perfect square is 1.

(d) We have 9800.

Let us find the square root.

	98
9	9800
	81
188	1700
	1504
	196

Clearly, $98^2 < 9800 < 99^2$

$99^2 = 99 \times 99 = 9801$.

Thus, the required least number to be added to 9800 to make it a perfect square is 1.

7. Number of plants in each column = $\sqrt{202500}$

Let us find the square root of 202500.

	450
4	202500
	16
85	425
	425
900	000
	000
	0

$\therefore \sqrt{202500} = 450$

Hence, there are 450 plants in each column.

8. 1 hectare = 10000 m^2.

$\therefore$ 81 hectares = 81 × 10000 = 810000 m^2

Let us first find the side of the square field.

side = $\sqrt{area}$ = $\sqrt{81000}$ m

$\therefore$ Side of the square field = 900 m.

Total length of the boundary of the square field.

= Circumference of the field = 4 × side = 4 × 900 m = 3600 m.

Hence, the total length of the boundary of the square field is 3600 m.

	900
9	810000
	81
180	000
	000
	0

$\therefore \sqrt{81000} = 900$

9. Total number of flowers in 10 baskets = 640 × 10 = 6400 flowers.

As Simran puts equal number of flowers in each temple as the number of temples as the number of temples in the city.

$\therefore$ Number of temples in the city = $\sqrt{6400}$ = 80.

Hence, there are 80 temples in the city.

10. Given that :

number of rows = number of soldiers in 1 row.

$\therefore$ Number of rows = $\sqrt{1600}$ = 40.

Hence, the number of rows of soldiers is 40.

	40
4	1600
	16
80	000
	000
	0

$\therefore \sqrt{1600} = 80.$

Exercise 3.4

1. (a) We have 9.9856. Let us find its square root.

	3.16
3	9.9856
	9
61	098
	61
626	3756
	3756
	0

$\therefore \sqrt{9.9856} = 3.16.$

Alternate method :

$9.9856 = \frac{99856}{10000}$

Now, $\sqrt{99856} = 316$

and $\sqrt{10000} = 100$

$\therefore \frac{\sqrt{99856}}{\sqrt{10000}} = \frac{316}{100} = 3.16.$

(b) Let us find its square root of 1.2544.

	1.12
1	1.121.2544
	1
21	025
	21
222	444
	444
	0

$\therefore \sqrt{1.2544} = 1.12.$

(c) Let us find its square root of 3696.64.

	60.8
6	3696.64
	36
1208	09664
	9664
	0

$\therefore \sqrt{3696.64} = 60.8.$

(d) Let us find its square root of 147.1369.

	12.13
1	147.1369
	1
22	47
	44
241	313
	241
2423	7269
	7269
	0

$\therefore \sqrt{147.1369} = 12.13.$

2. (a) We can write 5 as 5.000000.

	2.236
2	5.000000
	4
42	100
	84
443	1600
	1329
4466	27100
	26796
	404

$\therefore \sqrt{5} = 2.236.$

(b) We can write 2 as 2.000000.

	1.414
1	2.000000
	1
24	100
	96
281	400
	281
2824	11900
	11296
	604

$\therefore \sqrt{2} = 1.414.$

(c) We can write 6.5 as 6.50000.

	2.549
2	6.500000
	4
45	250
	225
504	2500
	2016
5089	48400
	45801
	2599

$\therefore \sqrt{6.5} = 2.549.$

(d) We can write 175 as 175.0000.

	13.22
1	175.000000
	1
27	075
	69
262	600
	524
2642	7600
	5284
	2316

$\therefore \sqrt{175} = 13.22.$

3. (a) $\sqrt{\frac{289}{144}} = \frac{\sqrt{289}}{\sqrt{144}}$

	17
1	289
	1
27	189
	189
	0

	12
1	144
	1
22	044
	44
	0

$\therefore \sqrt{\frac{289}{144}} = \frac{\sqrt{289}}{\sqrt{144}} = \frac{17}{12}$.

(b) $\sqrt{\frac{22500}{169}} = \frac{\sqrt{22500}}{\sqrt{169}}$

	150
1	22500
	1
25	125
	125
	0

	13
1	169
	1
23	069
	69
	0

$\therefore \sqrt{22500} = 150$ and $\sqrt{169} = 13$

Now $\frac{\sqrt{22500}}{\sqrt{169}} = \frac{150}{13}$ $\sqrt{\frac{22500}{169}} = \frac{150}{13}$

(c) $\sqrt{1\frac{800}{1225}} = \sqrt{\frac{2025}{1225}} = \frac{\sqrt{2025}}{\sqrt{1225}}$

	45
4	2025
	16
85	425
	425
	0

	35
3	1225
	9
65	325
	325
	0

$\therefore \sqrt{2025} = 45$ $\therefore \sqrt{1225} = 35$

Now $\frac{\sqrt{2025}}{\sqrt{1225}} = \frac{45}{35}$ $\therefore \sqrt{1\frac{800}{1225}} = \frac{45}{35}$.

(d) Similar work to be done.

4. (a) $\sqrt{81 \times 144} = \sqrt{81} \times \sqrt{144} = 9 \times 12 = 108$.

(b) $\sqrt{2.56 \times 5.29} = \sqrt{2.56} \times \sqrt{5.29}$

	1.6
1	2.56
	1
26	156
	156
	0

	2.3
2	5.29
	4
43	129
	129
	0

$\therefore \sqrt{2.56} = 1.6$ $\therefore \sqrt{5.29} = 2.3$

Now $\sqrt{2.56} \times \sqrt{5.29} = 1.6 \times 2.3 = 3.68$.

$\therefore \sqrt{2.56 \times 5.29} = 3.68$

(c) $\sqrt{0.0081 \times 256} = \sqrt{0.0081} \times \sqrt{256}$

	0.09
0	0.0081
	0
0	00
	00
09	81
	81
	0

	16
1	256
	1
26	156
	156
	0

$\therefore \sqrt{0.0081} = 0.09$ $\therefore \sqrt{256} = 16$

Now, $\sqrt{0.0081} \times \sqrt{256} = 0.09 \times 16 = 1.44$

$\therefore \sqrt{0.0081 \times 256} = 1.44$

(d) Similar work to be done.

5. We have $\sqrt{56205}$

Let us find its value.

	237.07
2	56205.0000
	4
43	162
	129
467	3305
	3269
47407	360000
	331849
	28151

$\therefore \sqrt{56205} = 237.07$

6. We have $\sqrt{1025}$

Let us find its value.

	32.015
3	1025.000000
	9
62	125
	124
6401	10000
	6401
64025	359900
	320125
	39775

$\therefore \sqrt{1025} = 32.015.$

7. (a)

	15
1	225
	1
25	125
	125
	0

$\therefore \sqrt{225} = 15.$

(b)

	23
2	550
	4
43	150
	129
	21

Clearly, $23^2 < 550 < 24^2$

$23^2 = 23 \times 23 = 529$

and $24^2 = 24 \times 24 = 576$

329 is more closer to 550 than 576.

Thus, $\sqrt{550}$ is approximately 23.

(c)

	13
1	178
	1
23	078
	69
	9

Clearly, $13^2 < 178 < 14^2$

$132 = 13 \times 13 = 169$ and $14^2 = 14 \times 14 = 196$

169 is more closer to 178 than 196.

Thus, the estimated square root of 178 is 13.

(d) Similar work to be done.

Revision Exercise

1. Similar to Q. 4 to Exercise 3.1.
2. Similar to Q. 3 to Exercise 3.1.
3. (a)

	58
5	3364
	25
108	864
	864
	0

$\therefore \sqrt{3364} = 58.$

Thus, the units place digit is 8.

(b)

	141
1	19881
	1
24	98
	96
281	281
	281
	0

$\therefore \sqrt{19881} = 141.$

Thus, the units place digit is 1.

(c) $\sqrt{1089} = 33$.

Thus, the units place digit is 3.

(d)

	902
9	813604
	81
1802	3604
	3604
	0

$\therefore \sqrt{813604} = 902$.

Thus, the units place digit is 2.

4. (a) We have 7225.

By prime factorisation,

5	7225
5	1445
17	289
17	17
	1

$7225 = 5 \times 5 \times 17 \times 17 \quad \therefore \sqrt{7225} = 5 \times 17 = 85$.

(b) We have 1024.

By prime factorisation,

$1024 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$\therefore \sqrt{1024} = 2 \times 2 \times 2 \times 2 \times 2 = 32$.

(c) Similar work to be done.

(d) Similar work to be done.

5. (a)

	222
2	49284
	4
42	92
	84
442	884
	884
	0

$\therefore \sqrt{49294} = 222$.

(b)

	96
9	9216
	81
186	1116
	1116
	0

$\therefore \sqrt{9216} = 96$.

(c)

	124
1	15376
	1
22	53
	44
244	976
	976
	0

$\therefore \sqrt{15376} = 124.$

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

(d)

	146
1	21316
	1
24	113
	96
286	1716
	1716
	0

$\therefore \sqrt{21316} = 146.$

6. (a) $\sqrt{\frac{484}{625}} = \frac{\sqrt{484}}{\sqrt{625}}$

	22
2	484
	4
42	84
	84
	0

	25
2	625
	4
45	225
	225
	0

$\therefore \sqrt{484} = 22.$ $\therefore \sqrt{625} = 25.$

Now $\frac{\sqrt{484}}{\sqrt{625}} = \frac{22}{25}$

$\therefore \sqrt{\frac{484}{625}} = \frac{22}{25}$

(b) $\sqrt{81 \times 121} = \sqrt{81} \times \sqrt{121} = 9 \times 11 = 99.$

(c) $\sqrt{1\frac{35}{289}} = \sqrt{\frac{324}{289}} = \frac{\sqrt{324}}{\sqrt{289}}$

	18
1	324
	1
28	224
	224
	0

	17
1	289
	1
27	189
	189
	0

$\therefore \sqrt{324} = 18.$ $\therefore \sqrt{289} = 17.$

Now $\dfrac{\sqrt{324}}{\sqrt{289}} = \dfrac{18}{17} \therefore \sqrt{1\dfrac{35}{289}} = \dfrac{18}{17}$.

(d) Similar work to be done.

7. (a)

	25.5
2	650.25
	4
45	250
	225
505	2525
	2525
	0

$\therefore \sqrt{650.25} = 25.5$

(b)

	2.34
2	5.4756
	4
43	147
	129
464	1856
	1856
	0

$\therefore \sqrt{5.4756} = 2.34.$

(c)

	0.006
0	0.000036
	0
0	00
6	36
	36
	0

$\therefore \sqrt{0.000036} = 0.006.$

(d) Similar work to be done.

8. (a)

	2.44
2	6.0000
	4
44	200
	176
484	2400
	1936
	464

$\therefore \sqrt{6} = 2.44.$

(b)

	3.31
3	11.0000
	9
63	200
	189
661	1100
	661
	439

$\therefore \sqrt{11} = 3.31.$

(c)

	15.93
1	254.0000
	1
25	154
	125
309	2900
	2781
3183	11900
	9549
	2351

$\therefore \sqrt{254} = 15.93.$

(d)

	13.41
1	180.0000
	1
23	80
	69
264	1100
	1056
2681	4400
	2681
	1719

$\therefore \sqrt{180} = 13.41.$

9. The greatest 6-digit number is 999999.

   It is not a perfect square $999^2 < 999999 < 1000^2$

   $999^2 = 999 \times 999 = 998001$

   $1000^2 = 1000000$, which is a 7-digit number.

   Thus, the greatest 6-digit perfect square number is 998001.

	999
9	999999
	81
189	1899
	1701
1989	19800
	17901
	1899

10. Let us find the square root of 40375.

	200
2	40375
	4
400	0375
	0
	375

   Clearly, $200^2 < 40375 < 201^2$.

   $201^2 = 201 \times 201 = 40401$

   Now, 40401 – 40375 = 26

   Thus, the required number to be added is 26.

11. Similar work to be done.

12. Let us try to find the square root of 2053.

	45
4	2053
	16
85	453
	425
	28

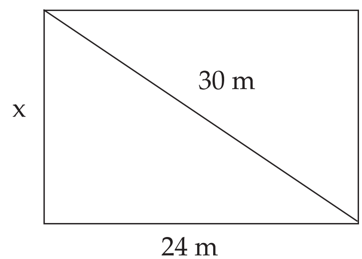
Clearly, $45^2 < 2053 < 46^2$.

$46^2 = 46 \times 46 = 2116$

Now, $2116 - 2053 = 63$

Hence, the minimum number of plants needed is 63 plants.

13. The given figure shows the rectangular hall.



Let the breadth of the rectangular hall be x m.

Then by pythagoras theorem,

$(30\text{ m})^2 = x^2 + (24\text{ m})^2$

$x^2 = (30\text{ m})^2 - (24\text{ m})^2 = 900\text{ m}^2 - 576\text{ m}^2 = 324\text{ m}^2$

$\therefore\ x = \sqrt{324}\text{m} = 18\text{ m}.$

Hence, the breadth of the rectangular hall is 18 m.

14. The required decimal fraction is the square root of 0.813604.

	0.902
9	0.813604
	81
1802	3604
	3604
	0

$\therefore\ \sqrt{0.813604} = 0.902.$

Hence, the required decimal fraction is 0.902.

15. Finding the square root of 2025 and 1849.

	45
4	2025
	16
85	425
	425
	0

	43
4	1849
	16
83	249
	249
	0

$\therefore\ \sqrt{2025} = 45.$ $\qquad \therefore\ \sqrt{1849} = 43.$

Now, $\dfrac{\sqrt{0.2025} + \sqrt{0.1849}}{\sqrt{0.2025} \quad \sqrt{0.1849}} = \dfrac{0.45 + 0.43}{0.45 - 0.43} = \dfrac{0.88}{0.02} = \dfrac{88}{2} = 44.$

Multiple Choice Questions

1. A perfect square never ends in 7, 2 and 3. Thus, the correct option is (d).

2. The units place digit of a perfect square can never be 2. Thus, the correct option is (c).

3. The number of digits in the square root of a 6-digit number is three. Thus, the correct option is (c).

4. See Answers given in the book.

5. If $\sqrt{17424} = 132$, then $\sqrt{174.24} = 13.2$. Thus, the correct option is (c).

6. Given : $\sqrt{15625} = 125$

   $\therefore \sqrt{156.25} - \sqrt{1.5625} = 12.5 - 1.25 = 11.25.$

   Thus, the correct option is (d).

7. $\sqrt{54 - \sqrt{21 + \sqrt{19 - 19}}} = \sqrt{54 - \sqrt{21 + \sqrt{19 - 3}}} = \sqrt{54 - \sqrt{21 + \sqrt{16}}}$

   $= \sqrt{54 - \sqrt{21 + 4}} = \sqrt{54 - \sqrt{25}} = \sqrt{54 - 5} = \sqrt{49} = 7.$

   Thus, the correct option is (b).

8. Perfect square between 1 and 100 are : 4, 9, 16, 25, 36, 49, 64 and 81. That is, there are eight perfect squares between 1 and 100. Thus, the correct option is (b).

9. As $2 \times 2 = 4$, $8 \times 8 = 64$ and $1 \times 1 = 1$, so 101^2 would not have the unit place digit as 4.

   Thus, the correct option is (d).

10. $\sqrt{2401} = 49$, $\sqrt{2025} = 45$, $\sqrt{1849} = 43$ and $\sqrt{2116} = 46$.

    Thus, the correct option is (d).

Mental Maths

A. See the Answers given in the book.

B. See the Answers given in the book.

Higher Order Thinking Skills (HOTS)

1. Perimeter of a square = 72 m [Given]

   $4 \times \text{sides} = 72$ m

   side = 72 m ÷ 4 = 18 m

   $\therefore$ Area of the square = side × side = 18 m × 18 m = 324 m^2.

2. $\sqrt{0.02 \times 0.2 \times a} = 0.2 \times 0.02 \times \sqrt{b}$

   $\sqrt{0.02 \times 0.2} \times \sqrt{a} = 0.004 \times \sqrt{b}$

   $\dfrac{\sqrt{a}}{\sqrt{b}} = \dfrac{0.004}{\sqrt{0.004}}$

   $\dfrac{a}{b} = \dfrac{0.000016}{0.004} = 0.004.$

3. (i) $\sqrt{0.014641} + \sqrt{146.41} + \sqrt{1.4641} = 0.121 + 12.1 + 1.21 = 13.431$

   (ii) $\sqrt{14641} = 121.$

4 Cubes and Cube Roots

Exercise 4.1

1. (a) $9^3 = 9 \times 9 \times 9 = 729$

(b) $(-15)^2 = (-15) \times (-15) \times (-15) = -3375$

(c) $(-6)^3 = (-6) \times (-6) \times (-6) = -216$

(d) $40^3 = 40 \times 40 \times 40 = 64000$

(e) $(0.6)^3 = 0.6 \times 0.6 \times 0.6 = 0.216$

(f) $\left(\frac{5}{7}\right)^3 = \frac{5}{7} \times \frac{5}{7} \times \frac{5}{7} = \frac{125}{343}$

(g) $\left(\frac{-3}{5}\right)^3 = \frac{-3}{5} \times \frac{-3}{5} \times \frac{-3}{5} = \frac{-27}{125}$

(h) $\left(\frac{-1}{4}\right)^3 = \frac{-1}{4} \times \frac{-1}{4} \times \frac{-1}{4} = \frac{-1}{64}$

2. (a) By prime factorisation,

13	1859
13	143
11	11
	1

$\therefore 1859 = 13 \times 13 \times 11$

As prime factors are not in triplet, so 1859 is not a perfect cube.

(b) By prime factorisation,

2	1728
2	864
2	432
2	276
2	108
2	54
3	27
3	9
3	3
	1

$\therefore 1728$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

As factors 2 and 3 are in triplets, so 1728 is a perfect cube.

(c) By prime factorisation,

2	21952
2	10976
2	5488
2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$\therefore 21952$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7 \times 7$

As the factors are in triplets, so 21952 is a perfect cube.

(d) By prime factorisation,

2	2000
2	1000
2	500
2	250
2	125
5	25
5	5
	1

$\therefore 2000 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5$

As 2 is not in triplets, so 2000 is not a perfect cube.

3. By prime factorisation,

2	12500
2	6250
5	3125
5	625
5	125
5	25
5	5
	1

$\therefore$ 12500 = 2 × 2 × 5 × 5 × 5 × 5 × 5

Factors 2 and 5 are not occurring in triplets.

To make triplets, we need one 2 and one 5.

Thus, the required number by which 12500 should be multiplied is 2 × 5 = 10.

4. By prime factorisation,

2	108000
2	54000
2	27000
2	13500
2	6750
3	3375
3	1125
3	375
5	125
5	25
5	5
	1

$\therefore$ 108000

= 2 × 2 × 2 × 2 × 2 × 3 × 3 × 3 × 5 × 5 × 5

Here, the factor 2 is not making a triplet.

Thus, the required number by which 108000 must be divided is 2 × 2 = 4.

5. By prime factorisation,

2	43200
2	21600
2	10800
2	5400
2	2700
2	1350
3	675
3	225
3	75
5	25
5	5
	1

$\therefore$ 43200 = 2 × 2 × 2 × 2 × 2 × 2 × 3 × 3 × 3 × 5 × 5

Here, the factor 5 is not occurring in triplet.

So, 43200 is not a perfect cube. To make it a perfect cube, we should multiply 43200 by 5.

Thus, the required least number is 5.

6. By prime factorisation,

2	21296
2	10648
2	5324
2	2662
11	1331
11	121
11	11
	1

$\therefore$ 21296 = 2 × 2 × 2 × 2 × 11 × 11 × 11

Here, the factor 2 is not occurring in triplet.

So, 21296 is not a perfect cube.

To make 21296 a perfect cube, we should divide it by 2.

Thus, the required least number is 2.

Exercise 4.2

1. (a) We have 3375. The number ends in 5, so ones place digit of its cube root will be 5.

$\sqrt[3]{3375}$

For tens digit : $1^2 < 3 < 2^2$

Thus, $\sqrt[3]{3375} = 15$.

(b) We have 42875.

$\sqrt[3]{42875}$

The number ends in 5, so the units place digit of it cube root will be 5.

For tens digit : $3^3 < 42 < 4^3$

Thus, $\sqrt[3]{42875} = 35$.

(c) We have 21952.

$\sqrt[3]{21952}$

For units digit: The number ends in 2, so the units place digit of its cube root will be 8.

For tens digit : $2^3 < 21 < 3^3$

Thus, $\sqrt[3]{21952} = 28$.

(d) Similar work to be done.

2. (a) By prime factorisation,

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

$\therefore$ 46656

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$\therefore \sqrt[3]{46656} = 2 \times 2 \times 3 \times 3 = 36$

(b) By prime factorisation,

2	74088
2	37044
2	18522
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$\therefore$ 74088

$= 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7$

$\therefore \sqrt[3]{74088} = 2 \times 3 \times 7 = 42$

c) By prime factorisation,

3	35937
3	11979
3	3993
11	1331
11	121
11	11
	1

$\therefore 35937 = 3 \times 3 \times 3 \times 11 \times 11 \times 11$

$\therefore \sqrt[3]{35937} = 3 \times 11 = 33$.

(d) Similar work to be done.

3. (a) $\sqrt[3]{\dfrac{729}{1000000}} = \dfrac{\sqrt[3]{729}}{\sqrt[3]{1000000}} = \dfrac{\sqrt[3]{9 \times 9 \times 9}}{\sqrt[3]{100 \times 100 \times 100}} = \dfrac{9}{100}.$

(b) $\sqrt[3]{27000} = \sqrt[3]{27 \times 1000} = \sqrt[3]{27} \times \sqrt[3]{1000} = \sqrt[3]{3 \times 3 \times 3} \times \sqrt[3]{10 \times 10 \times 10} = 3 \times 10 = 30.$

(c) $\sqrt[3]{512 \times 343} = \sqrt[3]{512} \times \sqrt[3]{343} = \sqrt[3]{8 \times 8 \times 8} \times \sqrt[3]{7 \times 7 \times 7} = 8 \times 7 = 56.$

(d) $\sqrt[3]{1000 \times 0.125} = \sqrt[3]{1000} \times \sqrt[3]{0.125} = \sqrt[3]{10 \times 10 \times 10} \times \sqrt[3]{0.5 \times 0.5 \times 0.5} = 10 \times 0.5 = 5.$

(e) $\sqrt[3]{0.001331} = \sqrt[3]{\dfrac{1331}{1000000}} = \dfrac{\sqrt[3]{11 \times 11 \times 11}}{\sqrt[3]{100 \times 100 \times 100}} = \dfrac{11}{100} = 0.11.$

(f) $\sqrt[3]{\dfrac{0.0008}{100}} = \sqrt[3]{\dfrac{8}{100 \times 10000}} = \dfrac{\sqrt[3]{8}}{\sqrt[3]{1000000}} = \dfrac{\sqrt[3]{2 \times 2 \times 2}}{\sqrt[3]{100 \times 100 \times 100}} = \dfrac{2}{100} = 0.02.$

(g) $\sqrt[3]{-343} = \sqrt[3]{-1} \times \sqrt[3]{343} = \sqrt[3]{(-1) \times (-1) \times (-1)} \times \sqrt[3]{7 \times 7 \times 7} = -1 \times 7 = -7.$

(h) $\sqrt[3]{\dfrac{-0.064}{27}} = \dfrac{\sqrt[3]{-0.064}}{\sqrt[3]{27}} = \dfrac{\sqrt[3]{-1 \times 0.064}}{\sqrt[3]{27}}$

$$= \frac{\sqrt[3]{(-1) \times (-1) \times (-1)} \times \sqrt{(0.4) \times (0.4) \times (0.4)}}{\sqrt{3 \times 3 \times 3}}$$

$$= \frac{-1 \times 0.4}{3} = \frac{-0.4}{3} = \frac{-4}{30} = \frac{-2}{15}.$$

4. Volume of the cube = side × side × side = 2.8 m × 2.8 m × 2.8 m = 21.952 m^3.

5. Let the numbers be 2x, 3x and 4x.

Then $(2x)^3 + (3x)^3 + (4x)^3 = 792$

$\Rightarrow 8x^3 + 27x^3 + 64x^3 = 792$

$\Rightarrow 99x^3 = 792$

$\Rightarrow x^3 = 792 \div 99 = 8$

$\Rightarrow x^3 = 2^3$

$\Rightarrow x = 2.$

$\therefore$ 2 x = 2 × 2 = 4, 3 x = 3 × 2 = 6 and 4 x = 4 × 2 = 8.

Hence, the numbers are 4, 6 and 8.

6. Volume of a cube = 17.576 m^3

$\Rightarrow$ side × side × side = 17.576 m^3

$\Rightarrow$ $side^3 = 17.576\ m^3$

$\Rightarrow$ side = $\sqrt[3]{17.576}$ m

$$= \sqrt[3]{\frac{17576}{1000}}\ m = \frac{\sqrt[3]{2 \times 2 \times 2 \times 13 \times 13 \times 13}}{\sqrt[3]{10 \times 10 \times 10}}\ m$$

$$= \frac{2 \times 13}{10}\ m = \frac{26}{10}\ m = 2.6\ m.$$

2	17576
2	8788
2	4394
13	2197
13	169
13	13
	1

Hence, the side of the cube is 2.6 m.

7. By prime factorisation,

2	8788
2	4394
13	2197
13	169
13	13
	1

$\therefore$ 8788 = 2 × 2 × 13 × 13 × 13

Here, factor 2 is not occurring in triplet. To make 8788 a perfect cube, it must be multiplied by 2.

$\therefore$ 8788 × 2 = 17576, which is a perfect cube.

$\therefore$ Cube root of 17576 = 2 × 13 = 26.

8. (a) By prime factorisation,

3	6591
13	2197
13	169
13	13
	1

$\therefore$ 6591 = 2 × 13 × 13 × 13

Here, factor 2 is not occurring in triplet.

To make 6591 a perfect cube, we should divide it by 3.

6591 ÷ 3 = 2197, which is a perfect cube.

$\sqrt[3]{2197} = 13$

(b) By prime factorisation,

2	5000
2	2500
2	1250
5	625
5	125
5	25
5	5
	1

$\therefore$ 5000 = 2 × 2 × 2 × 5 × 5 × 5 × 5

Here, factor 5 is not occurring in triplet.

To make 5000 a perfect cube, it should divided by 5.

5000 ÷ 5 = 1000

$\sqrt[3]{1000} = 10.$

(c) By prime factorisation,

2	6750
3	3375
3	1125
3	375
5	125
5	25
5	5
	1

$\therefore$ 6750 = 2 × 3 × 3 × 3 × 5 × 5 × 5

Here, the factor 2 is not occurring in triplet.

So, 6750 must be divided by 2 make it a perfect cube. 6750 ÷ 2 = 33756, which is a perfect cube.

Now, $\sqrt[3]{375} = 3 \times 5 = 15.$

Revision Exercise

1. (a) By prime factorisation,

2	8000
2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
5	5
	1

$\therefore$ 8000 = 2 × 2 × 2 × 2 × 2 × 2 × 5 × 5 × 5

$\therefore \sqrt[3]{8000} = 2 \times 2$

(b) By prime factorisation,

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$\therefore$ 128 = 2 × 2 × 2 × 2 × 2 × 2 × 2

As the factor 2 does not form a triplet, So 128 is not a perfect cube.

(c) By prime factorisation,

19	6859
19	361
19	19
	1

$\therefore$ 6859 = 19 × 19 × 19

$\sqrt[3]{6859} = 19$

As no factor is is left, 6859 is perfect triplet.

(d) Similar work to be done.

2. (a) Cube of 7.5 = 7.5 × 7.5 × 7.5

$$= \frac{75}{10} \times \frac{75}{10} \times \frac{75}{10} = \frac{421875}{1000} = 421.875$$

(b) Cube of $\frac{1}{4}$ = 0.25 × 0.25 × 0.25

$$= \frac{25}{100} \times \frac{25}{100} \times \frac{25}{100} = \frac{15625}{1000000} = 0.015625$$

(c) Cube = 38 = 38 × 38 × 38 = 54872

(d) Similar work to be done.

3. (a) We have 2744.

$\sqrt[3]{2744}$

T U — For units digit : The number ends in 4, so the units digit in its cube root will be 4.

For tens digit. $1^2 < 2 < 2^2$

Thus, $\sqrt[3]{2744} = 14$

(b) We have 2197.

$\sqrt[3]{2197}$

T U — For units digit : The number ends in 7, so the units digit in its cube root will be 3.

For tens digit. $1^2 < 2 < 2^2$

Thus, $\sqrt[3]{2197} = 13$

(c) Similar work to be done.

(d) Similar work to be done.

4. (a) By prime factorisation,

3	729
3	243
3	81
3	27
3	9
3	3
	1

$\therefore 729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$\therefore \sqrt[3]{729} = 3 \times 3 = 9.$

(b) By prime factorisation,

2	17576
2	8788
2	4394
13	2197
13	169
13	13
	1

$\therefore 17576 = 2 \times 2 \times 2 \times 13 \times 13 \times 13$

$\therefore \sqrt[3]{17576} = 2 \times 13 = 26.$

(c) Similar work to be done.

(d) Similar work to be done.

5. (a) We have $\sqrt[3]{3.375}$

$$3.375 = \frac{3375}{1000} = \frac{15 \times 15 \times 15}{10 \times 10 \times 10}$$

$$\therefore \sqrt[3]{3.375} = \frac{15}{10} = 1.5.$$

(b) We have $\sqrt[3]{2.197 \times 6.859} = \sqrt[3]{\frac{2197}{1000} \times \frac{6859}{1000}} = \sqrt[3]{\frac{2197}{1000}} \times \sqrt[3]{\frac{6859}{1000}}$

$2197 = 13 \times 13 \times 13$

$6859 = 19 \times 19 \times 19$

$\therefore \sqrt[3]{\frac{2197}{1000}} \times \sqrt[3]{\frac{6859}{1000}} = \frac{13}{10} \times \frac{19}{10} = 1.3 \times 19 = 2.47.$

Thus, $\sqrt[3]{2.197 \times 6.859} = 2.47.$

(c) $\sqrt[3]{\frac{0.001}{0.125}} = \sqrt[3]{\frac{1}{125}} = \sqrt[3]{\frac{1 \times 1 \times 1}{5 \times 5 \times 5}} = \frac{1}{5}.$

(d) Similar work to be done.

6. By prime factorisation,

2	58320
2	29160
2	14580
2	7290
3	3645
3	1215
3	405
3	135
3	45
3	15
5	5
	1

$\therefore 58320 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 5$

Here, factors 2 and 5 do not form triplets.

Thus, the required number by which 58320 must be divides is $2 \times 5 = 10$.

7. By prime factorisation,

5	8575
5	1715
7	343
7	49
7	7
	1

$\therefore 8575 = 5 \times 5 \times 7 \times 7 \times 7$

Here, the factor 5 does not a triplet.

Thus, the required by 8575 should be multiplied to make it a perfect cube is 5.

8. Let the numbers be x, 2x and 3x.

Then $x^3 + 8x^3 + 27x^3 = 26244$

$36\ x^3 = 26244$

$36\ x^3 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$36\ x^3 = 36 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$x^3 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$\sqrt[3]{x^3} = 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$x = 3 \times 3 = 9$

$2x = 2 \times 9 = 18$ and $3x = 3 \times 9 = 27$

Hence, the required numbers are 9, 18 and 27.

2	26244
2	13122
3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

9. Volume of a cube $= 0.216 = \dfrac{216}{1000}\text{m}^3$

$$\text{side} \times \text{side} \times \text{side} = \frac{216}{1000}\text{m}^3$$

$$\text{side}^3 = \frac{216}{1000}\text{m}^3$$

$$\text{side}^3 = \frac{6 \times 6 \times 6}{10 \times 10 \times 10}\text{m}^3$$

$$\text{side} = \sqrt[3]{\frac{6 \times 6 \times 6}{10 \times 10 \times 10}} = \frac{6}{10} = 0.6\ \text{m}^3.$$

10. Length of side of a cubical box = 0.5 m = 50 cm

$\therefore$ Volume of the box = side × side × side

= 50 cm × 50 cm × 50 cm = 125000 cm^3.

11. Required number of small cubes $= \dfrac{\text{Volume of cubical box}}{\text{Volume of 1 small cube}}$

$$= \frac{\cancel{400}^{20}\text{ cm} \times \cancel{400}^{20}\text{ cm} \times \cancel{400}^{20}\text{ cm}}{\cancel{20}\text{ cm} \times \cancel{20}\text{ cm} \times \cancel{20}\text{ cm}}$$

$$= 20 \times 20 \times 20 = 8000 \text{ cubes}$$

Hence, 8000 small cubes can be accommodated in the cubical box.

12. $\left(\sqrt{(14.3)^2 + (2.4)^2}\right)^3 = \left(\sqrt{204.49 + 5.76}\right)^3 = \left(\sqrt{210.25}\right)^3 = (14.5)^3 = 3048.625.$

13. Similar work to be done as Q.7. of this Exercise.

Multiple Choice Questions

1. The cube of a positive integer is positive and that of a negative number is negative. It means the cube of an integer may be positive or negative. Thus, the correct option is (d).
2. Perfect cubes between 1 and 100 are : 8, 27 and 64. That is, there are three perfect cubes between 1 and 100. Thus, the correct option is (d).
3. The cube of a number ending in 7 will end in 3. Thus, the correct option is (a).
4. The cube root of a number ending in 7 will end in 3. Thus, the correct option is (a).

5. See the **Answers** given in the book.

6. If the square of number ends with 6, so the units place digit of the number can be 4 or 6.
$4^3 = 4 \times 4 \times 4 = 64$ and $6^3 = 6 \times 6 \times 6 = 216$
$\therefore$ The cube of number will end with 4 or 6.
Thus, the correct option is (c).

7. Side of the cube = $\sqrt[3]{\text{Volume}}$

$$= \sqrt[3]{3.375} = \sqrt[3]{\frac{3375}{1000}} = \sqrt[3]{\frac{15 \times 15 \times 15}{10 \times 10 \times 10}} = \frac{15}{10} = 1.5 \text{ cm}$$

Thus, the correct option is (a).

8. Cube of $20 \times 20 \times 20 = 8000$. Thus, the correct option is (d).

9. The value of $\left\{(-1)^3\right\}^{37} = (-1)^{3 \times 37} = (-1)^{111} = -1$. Thus, the correct option is (c).

10. $$\frac{1}{\frac{\sqrt[3]{3.375}}{\sqrt[3]{0.125}}} = \frac{\sqrt[3]{0.125}}{\sqrt[3]{3.375}} = \frac{\sqrt[3]{0.5 \times 0.5 \times 0.5}}{\sqrt[3]{1.5 \times 1.5 \times 1.5}} = \frac{0.5}{1.5} = \frac{5}{15} = \frac{1}{3}.$$

Thus, the correct option is (c).

Mental Maths

A. See the Answers given in the book.

B. See the Answers given in the book.

Higher Order Thinking Skills (HOTS)

1. Yes, it is true if both x and y are 1.
$x = 1$ and $y = 1$.
$x \div y = 1 \div 1 = 1$
Also, $1^3 \div 1^3 = 1$.

2. $$\frac{\text{Volume of one cubical box}}{\text{Volume of another cubical box}} = \frac{1.728 \text{ m}^3}{3375000 \text{ cm}^3} = \frac{1728\cancel{000} \text{ cm}^3}{3375\cancel{000} \text{ cm}^3} = \frac{1728}{3375}$$

$$= \frac{\text{side}_1 \times \text{side}_1 \times \text{side}_1}{\text{side}_2 \times \text{side}_2 \times \text{side}_2} = \frac{1728}{3375}$$

$$= \frac{\text{side}_1}{\text{side}_2} = \frac{\sqrt[3]{1728}}{\sqrt[3]{3375}} = \frac{\sqrt[3]{12 \times 12 \times 12}}{\sqrt[3]{15 \times 15 \times 15}} = \frac{\cancel{12}^4}{\cancel{15}_5} = \frac{4}{5}$$

Hence, ratio of their sides is 4 : 5.

3. Volume of the hall = $40 \text{ m} \times 20 \text{ m} \times 10 \text{ m} = 8000 \text{ m}^3$.
As same material is used in constructing the cube, so their volumes will be equal.
$\therefore$ Volume of the cube = 8000 m^3
side × side × side = 8000 m^3 [$\because$ Sides of a cube are equal.]
side = $\sqrt[3]{8000} = \sqrt[3]{20 \times 20 \times 20} = 20$ m.
Hence, the dimensions of the cube are 20 cm × 20 cm × 20 cm.

5 Playing with NumbersS

Exercise 5.1

1. (a) $54 = 5 \times 10 + 4$ (b) $38 = 3 \times 10 + 8 = 10 \times 3 + 8$
   (c) $259 = 2 \times 100 + 5 \times 10 + 9 = 100 \times 2 + 10 \times 5 + 9$
   (d) $786 = 7 \times 100 + 8 \times 10 + 6 = 100 \times 7 + 10 \times 8 + 6$

2. (a) $10 \times 3 + 2 = 30 + 2 = 32$
   (b) $10 \times 8 + 3 = 80 + 3 = 83$
   (c) $100 \times 4 + 10 \times 2 + 6 = 400 + 20 + 6 = 426$
   (d) $100 \times 8 + 10 \times 1 + 5 = 800 + 10 + 5 = 815$

3. (a)
$$\begin{array}{ccc} & B & 6 \\ & +5 & A \\ 1 & 3 & 4 \end{array}$$
As $6 + 8 = 14$, So $A = 8$.
Now $1 + B + 5 = 13$
$B = 13 - 6 = 7$
Thus, $A = 8$ and $B = 7$.

(b)
$$\begin{array}{ccc} 4 & 5 & 9 \\ +3 & A & B \\ A & 4 & 5 \end{array}$$
Clearly, $B = 6$ and $9 + 6 = 15$.
Now $1 + 5 + A = 14$
$A = 14 - 6 = 8$. Also $1 + 4 + 3 = 8$.
Thus, $A = 8$ and $B = 6$.

(c)
$$\begin{array}{ccc} 2 & 8 & A \\ +3 & B & 6 \\ A & 5 & 4 \\ \hline 9 & 8 & 3 \end{array}$$
Clearly, $A = 3$ as $3 + 6 + 4 = 13 = 10 + 3$
Now $1 + 8 + B + 5 = 14 + B$
Clearly, $B = 4$ as $14 + 4 = 18$
Again $= 2 + 3 + 3 + 1$ (Carried) $= 9$.
Thus, $A = 3$ and $B = 4$.

4. (a)
$$\begin{array}{cccc} 3 & A & 0 & 2 \\ +D & 8 & B & 6 \\ \hline 2 & 0 & C & B \\ \hline \end{array} \longrightarrow \begin{array}{cccc} 3 & A & 0 & 2 \\ +D & 8 & B & 6 \\ \hline 2 & 0 & C & B \\ \hline \end{array}$$
Thus, $A = 9$, $B = 6$, $C = 3$ and $D = 1$.

(b) We know that minuend – difference = subtract.
$\therefore 735 - 587 = 248$.
Thus,
$$\begin{array}{ccc} 7 & 3 & 5 \\ +2 & 4 & 8 \\ \hline 5 & 8 & 7 \\ \hline \end{array}$$

(c)
$$\begin{array}{cccc} 8 & 9 & 3 & 5 \\ +2 & 8 & 0 & 8 \\ \hline R & 2 & 0 & 5 \\ \hline \end{array} \longrightarrow \begin{array}{cccc} 8 & 0 & 3 & 5 \\ +2 & 8 & 2 & 8 \\ \hline 5 & 2 & 0 & 7 \\ \hline \end{array}$$
Thus, $P = 0$, $Q = 2$, $R = 5$ and $S = 7$.

5. (a)

$$\begin{array}{rrr} & B & 3 \\ & \times & 7 \\ \hline 3 & 0 & A \\ \hline \end{array}$$

Clearly A = 1 as 3 × 7 = 21

Also, B = 4 as 4 × 7 + 2 (Carried) = 30.

(b)

$$\begin{array}{rrrr} & 3 & 8 & A \\ & \times & 2 & A \\ 1 & B & 2 & A \\ 7 & 7 & 0 & \times \\ \hline B & 6 & 2 & A \\ \hline \end{array} \longrightarrow \begin{array}{rrrr} & 3 & 8 & 5 \\ & \times & 2 & 5 \\ 1 & 9 & 2 & 5 \\ 7 & 7 & 0 & \times \\ \hline 9 & 6 & 2 & 5 \\ \hline \end{array}$$

Thus, A = 5 and B = 9.

(c) See the Answers given in the book.

Exercise 5.2

1. 3 × 3 – 3 = 6

2. Four 4's can be arranged as (4 ÷ 4 + 4) × 4 = 20.

3. English alphabet are : A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
BOOK is written as ERRN
Here, the letter E is at third place from B, Q is as the third place from O and N is at the third place from K. Following this rule, we find that :
KH LV ILQH is HE IS FINE

4. Same rule as the Q.3 above is applied here.
Following this rule, we get : DQLPDO as ANIMAL.

5. (a) We have: 2, 3, 5, 9, 17, —, 65, —.
1 2 4 8
Here, the series has the rule 2n – 1.
Thus, 2, 3, 5, 9, 17, 33, 65, 129, ...

(b) We have 3 4, 6, 9, 13, 18, —, —.
1 2 3 4 5
Here, the difference is increased by 1.
Thus, 3, 4, 6, 9, 13, 18, 24, 31

6. The complete magic square is given below.

4	15	11
17	10	3
9	5	16

7. The complete magic square is given below.

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

8. See the **Answers** given in the book.

Exercise 5.3

1. We know that a number having units place digit as 0, 2, 4, 6 or 8 is divisible by 2.
   (a) As the units place digit of 3850 is 0, so it is divisible by 2.
   (b) As the units place digit of 1896 is 6, so it is divisible by 2.
   (c) As the units place digit of 309 is not an even number, so it is not divisible by 2.
   (d) As the units place digit of 45168 is 8, so it is divisible by 2.

2. We know that a number is divisible by 5 if its ones place digit is either 0 or 5.
   Also, a number is divisible by 10 if its ones place digit is 0.
   (a) 3850 is divisible by both 5 and 10.
   (b) 4056 is neither divisible by 5 nor by 10.
   (c) 1985 is divisible by 5 but not by 10.
   (d) 3005 is divisible by 5 but not by 10.

3. We know that a number is divisible by 3 if the sum of its digits is a multiple of 3.
   Also, a number is divisible by 9 it the sum of its digits is a multiple of 9.
   (a) Sum of digits of 285 = 2 + 8 + 5 = 15 which is a multiple of 3 but not of 9.
   So, 285 is divisible by 3 but not by 9.
   (b) Sum of digits of 4071 = 4 + 0 + 7 + 1 = 12, which is a multiple of 3 but not of 9.
   So, 4071 is divisible by 3 but not by 9.
   (c) Sum of digits of 891135 = 8 + 9 + 1 + 1 + 3 + 5 = 27, which is a multiple of both 3 and 9.
   So, 891135 is divisible by both 3 and 9.
   (d) Sum of digits of 6271 = 6 + 2 + 7 + 1 = 16, which is neither a multiple of 3 nor of 9.
   So, 6271 is not divisible by both 3 and 9.

4. (a) We have 7*615.
   Sum of its digits = 7 + 6 + 1 + 5 = 19
   The multiple of 9 next to 19 is 27.
   Now, 27 – 19 = 8.
   Thus, the required least number that must replace * is 8.
   (b) We have 439 * 9.
   Sum of its digits = 4 + 3 + 9 + 9 = 25
   The multiple of 9 next to 25 is 27.
   Now, 27 – 25 = 2.
   Thus, the required least number that must replace * is 2.
   (c) We have 5 * 8.
   Sum of its digits = 5 + 8 = 13.
   The multiple of 9 next to 13 is 18.
   Now, 18 – 13 = 5.
   Thus, the required least number that must replace * is 5.
   (d) We have 73 * 0.
   Sum of its digits = 7 + 3 + 0 = 10.
   The multiple of 9 next to 10 is 18.
   Now, 18 – 10 = 8.

5. The complete table is given below.

Number	Divisible by 2	Divisible by 5	Divisible by 9	Divisible by 10
281	×	×	×	×
6941	✓	×	×	×
87840	✓	✓	✓	✓
4386	×	✓	×	×
195	×	✓	×	×
280	✓	✓	×	✓

Revision Exercise

1. (a) $57 = 10 \times 5 + 7 = 5 \times 10 + 7$

   (b) $239 = 2 \times 100 + 3 \times 10 + 9$

   (c) $654 = 6 \times 100 + 5 \times 10 + 4$

   (d) $26 = 2 \times 10 + 6$

2. (a)

$$\begin{array}{r} 3\,4\,1\,A \\ 1\,9\,4\,8 \\ +\,A\,6\,B\,9 \\ \hline D\,C\,1\,9 \\ \hline \end{array} \longrightarrow \begin{array}{r} 3\,4\,1\,2 \\ 1\,9\,4\,8 \\ +\;2\,6\,5\,9 \\ \hline 8\,0\,1\,9 \\ \hline \end{array}$$

Thus, A = 2, B = 5, C = 0 and D = 8.

   (b)

$$\begin{array}{r} 5\,2\,0\,A \\ -\,C\,1\,A\,5 \\ \hline 1\,0\,B\,3 \\ \hline \end{array} \qquad \begin{array}{r} 5\,2\,0\,8 \\ -\;4\,1\,8\,5 \\ \hline 1\,0\,2\,3 \\ \hline \end{array}$$

Thus, A = 8, B = 2 and C = 4.

   (c)

$$\begin{array}{r} 5\,B\,A \\ \times\,C\,3 \\ 1\,5\,1\,2 \\ 1\,0\,0\,8\,\times \\ \hline 1\,1\,5\,9\,2 \\ \hline \end{array} \qquad \begin{array}{r} 5\,0\,4 \\ \times\,2\,3 \\ 1\,5\,1\,2 \\ 1\,0\,0\,8\,\times \\ \hline 1\,1\,5\,9\,2 \\ \hline \end{array}$$

Thus, A = 4, B = 0 and C = 2.

3. We have 3P5082.
   Sum of its digits = 3 + 5 + 0 + 8 + 2 = 18, which is a multiple of 9.
   Thus, the value of P is 0 and the number is 305082.

4. (a) The smallest digit that replaces A is 3 as 3 ÷ 2 leaves the reminder 1.

   (b) The required smallest digit is 2 as 2 ÷ 2 = 0

   (c) The required smallest digit is 6 as 6 ÷ 4 leaves the remainder 2.

   (d) The required smallest digit is 8 as 8 ÷ 5 leaves the remainder 3.

   (e) The required smallest number is 5 as 5 ÷ 5 leaves no remainder.

   (f) The required smallest number is 9 as 9 ÷ 8 leaves the remainder 1.

5. Similar work to be done as Q.3. of Exercise 5.2.

6. See the Answer given in the book.

7. We have:

23		
22		
27		25

Required number in the bottom row = 72 – (27 + 25) = 72 – 52 = 20. Required number in a diagonal = 72 – (23 + 25) = 72 – 48 = 24.
Required number in the other diagonal = 72 – (27 + 24) = 72 – 51 = 21.
Required number in the middle row = 72 – (23 + 24) = 72 – 46 = 26.
Similarly, We will fine the other missing number.
Thus, the complete magic square is :

23	28	21
22	24	26
27	20	25

8. We have:

4	15	14	1
	6		
		11	
16			13

Similar work to be done as Q.7 above.
Thus, the complete magic square is :

4	15	14	1
9	6	7	12
5	10	11	8
16	3	2	13

9. Here the number in the centre is the sum of squares other numbers.
$1^2 = 1$, $3^2 = 9$, $2^2 = 4$, $1 + 9 + 4 = 14$
Similarly, $5^2 + 3^2 + 4^2 = 25 + 9 + 16 = 50$. $\therefore$ A = 50 and $77 - (6^2 + 5^2) = 77 - 61 = 16$ which the square of 4. Thus, B = 4.

Multiple Choice Questions

1. See the **Answers** given in the book.

2. The number x is 14 as 14 ÷ 5 gives 4 as remainder. Also 14 is completely divisible by 2.
As ones digit in 14 is 4, so the correct option is (b).

3. We have abc = 487.
Here, a = 4, b = 8 and c = 7

$\therefore$ bac = 847 and cba = 784

$\therefore$ bac – cba = 847 – 784 = 63.

Thus, the correct option is (b).

4. The quotient will be the difference of x and y, i.e., x – y.
Thus, the correct option is (a).

5. Difference = 572 – 275 = 297.
Sum of the digits of 297 = 2 + 9 + 7 = 18, which is a multiple of both 3 and 9. So, the difference will be divisible by both 3 and 9.
Thus, the correct option is (c).

6. A number is divisible by 5 if its ones place digit is either 0 or 5. So, the ones digit of the number should be 6.
Thus, the correct option is (b).

7. Sum of the digits of 5034 = 5 + 0 + 3 + 4 = 12, which is divisible by 3.
Thus the given number 5034 is divisible by both 2 and 3. Thus, the correct option is (a).

8. A number divisible by 8 will be divisible by both 2 and 4. Thus, the correct option is (c).

9. A number is divisible by 8 if its lost three digits are zeros or the number formed by them is a multiple of 8.
The number formed by lost three digits of 987303x8 is 3x8.
308 ÷ 8 leaves the remainder 4.
318 ÷ 8 leaves the remainder 6.
328 ÷ 8 leaves the remainder 0.
So, the smallest value of x should be 0.
Thus, the correct option is (b).

10. The given statement is true. Thus, the correct option is (a).

Mental Maths

A. See the **Answers** given in the book.

B. See the **Answers** given in the book.

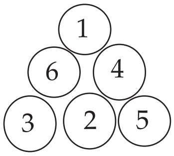
Higher Order Thinking Skills (HOTS)

1. Here, $\frac{1^2 + 2^2 + 3^2}{2} = \frac{1 + 4 + 9}{2} = \frac{14}{2} = 7.$

$\frac{5^2 + 6^2 + 7^2}{2} = \frac{25 + 36 + 49}{2} = \frac{110}{2} = 55.$

$\therefore A = \frac{7^2 + 8^2 + 9^2}{2} = \frac{49 + 64 + 81}{2} = \frac{194}{2} = 97.$

2. The required numbers are given in circles in the alongside figure.



6 Algebraic Expressions

Exercise 2.1

1. A polynomial is an algebraic expression in which the powers of the literal number are all whole numbers.

   (a) $\frac{2x^2}{3} - \frac{3}{4x^2} + 5$ is not a polynomial because the power of x in $\frac{3}{4x^2}$ is – 2 which is not a whole number.

   (b) $3x^3 - 4x + 9$ is a polynomial because the powers of variable x is all terms are whole numbers.

   (c) $\sqrt{2x} - 3x$ is not a polynomial because the power of x in $\sqrt{2x}$ is $\frac{1}{2}$.

2. (a) $3x^2 - 2y + 8$ is a trinomial as it has three terms

   (b) $x^2 + 9$ is a binomial as it has two terms.

   (c) $\frac{2}{5}$ abc is a monomial.

   (d) $7a^2 - 8b^2$ is a binomial.

3. The degree of a polynomial is the highest power of the variable.

   (a) The highest power of the variable is $4x^2 + 9x - y^3$ is 3. So its degree is 3.

   (b) The highest power f the variable is $2ab - 4a + b$ is 2(a'b'). So, its degree is 2.

   (c) The highest power of variable in $3a^2 + 2b^3 - ab$ is 3. So its degree is 3.

   (d) We have $x^3y^3 - 2x^2y + 2xy^2 - xy$.
   Power of variables xy in terms $x^3y^3 = 3 + 3 = 6$.
   Thus, the degree this polynomial is 6.

4. We know that like terms have some literal factors.

   (a) $2\ xy^2$ and $-3\ xy^2$ are like terms.

   (b) 4 ab and 4 a are not like terms.

   (c) $-x^2y^2z$ and x^2zy^2 are like terms.

   (d) In xy, x^2y, xy^2, xy^2, like terms xy^2, xy^2.

5. (a) $4a^2 - ab - c + a^2 + 2b - 6c$
   $= 4a^2 + a^2 = 2b + 2b - c - 6c$ [Putting the like terms together]
   $= 5\ a^2 - 7c$.

   (b) $6x^2 - 8\ y^3 - 5 + 6\ y^2 - 2\ x^2 + 8 = 6\ x^2 - 2\ x^2 - 8\ y^3 + 6\ y^2 - 5 + 8 = 4\ x^2 + 6\ y^2 - 8\ y^3 + 3$.

   (c) $2\ a^3 - 5\ ab + 9 + 6\ a^3 - 9 = 2\ a^3 + 6\ a^3 - 5\ ab + \cancel{9} - \cancel{9} = 8\ a^3 - 5\ ab$.

   (d) $7\ xy^2 - 2\ x^2y + xy + 4\ x^2y - xy + 5\ xy^2$
   $= 7\ xy^2 + 5\ xy^2 - 2\ x^2y + 4\ x^2y + \cancel{xy} - \cancel{xy}$ [Putting like terms together]
   $= 12\ xy^2 + 2\ x^2y$.

6. (a) $3\ y^2 - 8 + 6\ x^3 - (6\ x^3 - 2\ y^2 + 9)$
   $= 3\ y^2 - 8 + \cancel{6\ x^3} - \cancel{6\ x^3} + 2\ y^2 - 9$
   $= 3\ y^2 + 2\ y^2 - 8 - 9 = 5\ y^2 - 17$.

(b) $5a^2 - 8b^2 + 8c - (9a^2 - b^2 - c)$
$= 5a^2 - 8b^2 + 8c - 9a^2 + b^2 + c$
$= 5a^2 - 9a^2 + 8b^2 + b^2 + 8c + c$ [Putting like terms together]
$= -4a^2 - 7b^2 + 9c$

(c) $3xy - y - (15xy - 2y - 4)$
$= 3xy - y - 15xy + 2y + 4$
$= 3xy - 15xy - y + 2y + 4$
$= -12xy + y + 4$

(d) $6ab^2 + b^2 - 4a - (2ab^2 - b^2 + a)$
$= 6ab^2 + b^2 - 4a - 2ab^2 + b^2 - a$
$= 6ab^2 - 2ab^2 + b^2 + b^2 - 4a - a$
$= 4ab^2 + 2b^2 - 5a$.

7. Let the number to be added be A.
Then $A + 5x - xy + 9 = xy - 9x + 5x$
$\Rightarrow A = xy - 9x + 5x - (5x - xy + 9) = xy - 9x + \cancel{5x} - \cancel{5x} + xy - 9 = xy + xy - 9x - 9$
Hence, the required number to be added is $2xy - 9x - 9$.

8. To get the required number, we will subtract $4x^3 - 3x^2 + 8x - 5$ from $9x^3 + 6x^2 + x - 4$.
$\therefore 9x^3 + 6x^2 + x - 4 - (4x^3 - 3x^2 + 8x - 5)$
$= 9x^3 + 6x^2 + x - 4 - 4x^3 + 3x^2 - 8x + 5$
$= 9x^3 + 4x^3 + 6x^2 + 3x^2 + x - 8x - 4 + 5$
$= 5x^3 + 9x^2 - 7x + 1$
Hence, the required number to be subtracted is $5x^3 + 9x^2 - 7x + 1$.

9. Sum of $x^2 - y^2 + 3xy$ and $3x^2 - 3y^2 + 4xy$
$= x^2 - y^2 + 3xy + 3x^2 - 3y^2 + 4xy$
$= x^2 + 3x^2 - y^2 - 3y^2 + 3xy + 4xy = 4x^2 - 4y^2 + 7xy$.
Now $8x^2 - 5y^2 - 5xy - (4x^2 - 4y^2 + 7xy)$
$= 8x^2 - 5y^2 - 5xy - 4x^2 + 4y^2 - 7xy$
$= 8x^2 - 4x^2 - 5y^2 + 4y^2 - 5xy - 7xy$
$= 4x^2 - y^2 - 12xy$

10. Sum of $3a^2 - a^2 + 5a - 8$ and $5a - 6 + 4a^2$
$= 3a^2 - a^2 + 5a - 8 + 5a - 6 + 4a^2$
$= 2a^2 + 4a^2 + 5a + 5a - 8 - 6$
$= 6a^2 + 10a - 14$
Now $6a^2 + 10a - 14 + 2a + b - ab$
$= 6a^2 + 10a + 2a + b - ab - 14$
$= 6a^2 + 12a + b - ab - 14$

Exercise 6.2

1. (a) $4a \times (-3a^2) = -12a^3$.
(b) $5x^2 \times 2x^2 = 5 \times 2 \times x^{2+2} = 10x^4$.
(c) $5xy \times 6x^2y = 5 \times 6 \times x^{1+2} \times y^{1+1} = 30x^3y^2$.
(d) $-8ab \times (-5abc) = (-8) \times (-5) \times a^{1+1} \times b^{1+1} \times c = 40a^2b^2c$.

(e) $\frac{-2}{5}xy \times (-10\,xy^2) = \frac{-2}{\cancel{5}} \times \left(-\cancel{10}^{-2}\right) \times x^2y^3 = 4\,x^2y^3$

(f) $4\,xy \times 5\,x^2y = 4 \times 5 \times x^{1+2} \times y^{1+1} = 20\,x^3y^2$.

(g) $9x^2y \times 8\,xy^2 = 9 \times 8 \times x^{2+1} \times y^{1+2} = 72\,x^3y^3$

(h) $-abc \times (-5\,a^2b^2c^2) = (-1) \times (-5) \times a^{1+2} \times b^{1+2} \times c^{1+2} = 5\,a^3b^3c^3$.

2. (a) $2p \times (-3\,p^2) \times 6\,p^4 = 2 \times (-3) \times 6 \times p^{1+2+4} = -36\,p^7$.

(b) $xy \times (-3\,xy^2) \times x^2y = 1 \times (-3) \times 1 \times x^{1+1+2} \times y^{1+2+1} = -3\,x^4y^4$

(c) $8\,x \times (3\,xy) \times \frac{5}{8}xy^2 = \cancel{8} \times \frac{5}{\cancel{8}} \times x^{1+1+1} \times y^{1+2} = 5\,x^3y^3$

(d) $-3\,a^4 \times (-2\,a^2b^2) \times 4\,ab^2 = (-3) \times (-2) \times 4 \times a^{4+2+1} \times b^{2+2} = 24\,a^7b^4$.

3. (a) $(3\,x - 4) \times (2x + 5) = (3x - 4) \times 2x + (3x - 4) \times 5$

$= 6\,x^2 - 8\,x + 15\,x - 20 = 6\,x^2 - 7\,x - 20$

(b) $(2\,a + b)\,(4\,a - 3b) = (2a + b) \times 4a + (2\,a + b) \times (-3\,b)$

$= 8\,a^2 + 4\,ab - 6\,ab - 3\,b^2 = 8\,a^2 - 3\,b^2 - 2\,ab$

(c) $a^2\,(2\,a^3 + 9\,a^2 - a) = 2\,a^{2+3} + 9\,a^{2+2} - a^{2+1} = 2\,a^5 + 9\,a^4 - a^3$

(d) $\left(p + \frac{1}{3}q\right)\left(2p - \frac{2}{4}q\right) = \left(p + \frac{1}{3}q\right) \times 2p + \left(p + \frac{1}{3}q\right) \times \left(-\frac{2}{4}q\right)$

$= 2p^2 + \frac{2}{3}q + \left(\frac{1}{2}pq - \frac{1}{6}q^2\right) = 2p^2 + \frac{2}{3}pq - \frac{1}{2}pq - \frac{1}{6}q^2$

$= 2p^2 + \frac{4-3}{6}pq - \frac{1}{6}q^2 = 2p^2 + \frac{1}{6}pq - \frac{1}{6}q^2$

4. (a) $(3\,x^2 - 5x + 8) \times 2x = 6\,x^3 - 10\,x^2 + 16\,x$

(b) $(5\,x^2 - 3\,x + 7) \times (-4\,x^2) = -20\,x^4 + 12\,x^3 - 28\,x^2$

(c) $a^2\,(2\,a^3 + 9\,a^2 - a) = 2\,a^5 + 9\,a^4 - a^3$

(d) $\left(\frac{5x^3}{8} - \frac{x^2}{8} + \frac{1}{4}\right)\left(\frac{-2x}{5}\right) = \frac{-10x^4}{40} + \frac{2x^3}{10} - \frac{2x}{20} = \frac{-x^4}{4} + \frac{x^3}{5} - \frac{x}{10}$

5. (a) $(x^2 - y^2)\,(x^2 + y^2) = (x^2 - y^2)\,x^2 + (x^2 - y^2)\,y^2 = x^4 - \cancel{x^2y^2} + \cancel{x^2y^2} - y^4 = x^4 - y^4$

(b) $(a + b)\,(a^2 - ab + b^2) = a(a^2 - ab + b^2) + b\,(a^2 - ab + b^2)$

$= a^3 - \cancel{a^2b} + \cancel{ab^2} + \cancel{ba^2} - \cancel{ab^2} + b^3$

$= a^3 + b^3$

(c) $(x + y)\,(x^2 - y^2) = x\,(x^2 - y^2) + y\,(x^2 - y^2) = x^3 - xy^2 + yx^2 - y^3$

(d) $(2\,p + q)\,(2\,p^2 - 3\,q + 1) = 2\,p\,(2\,p^2 - 3q + 1) + q\,(2p^2 - 3\,q + 1)$

$= 4\,p^3 - 6\,pq + 2\,p + 2\,p^2q - 3\,q^2 + q$

6. (a)

$$\begin{array}{r} x - 3y \\ 3x + y \\ \hline xy - 3y^2 \\ 3x^2 - 9\,xy \\ \hline 3x^2 - 8\,xy - 3y^2 \\ \hline \end{array}$$

(b)

$$\begin{array}{r} 4a^2 - 4\ ab + b^2 \\ a - b \\ \hline - 4\ a^2b - 4\ ab^2 - b^3 \\ + 4a^3 - 4\ a^2b + ab^2 \\ \hline 4\ a^3 - 4\ a^2b - 4\ a^2b + 5\ ab^2 - b^3 \\ = 4\ a^3 - 8\ a^2b + 5\ ab^2 - b^3 \end{array}$$

[Multiplying by b]
[Multiplying by a]

(c)

$$\begin{array}{l} \frac{x}{4} \quad - \quad 7y \\ \frac{2x}{5} \quad + \quad \frac{y}{3} \\ \hline \frac{xy}{12} \quad - \quad \frac{7y^2}{3} \\ \frac{20x^2}{5} \quad \frac{14xy}{5} \\ \hline \frac{2x^2}{20} + \frac{xy}{12} - \frac{14xy}{5} - \frac{7y^2}{3} \\ \hline = \frac{x}{10} - \frac{163xy}{60} - \frac{7y^2}{3} \end{array}$$

[Multiplying by $\frac{y}{3}$]
[Multiplying by $\frac{2x}{5}$]
[Adding partial products]

(d)

$$\begin{array}{r} p^2 + 2\ p - 4 \\ p - 3\ q \\ \hline -3\ p^2q - 6\ pq + 12\ q \\ p^3 + 2\ p^2 - 4p \\ \hline p^3 - 3\ p^2q + 2\ p^2 - 6\ pq - 4\ p + 12\ q \end{array}$$

[Multiplying by –3q]
[Multiplying by p]

7. (a) Given : $x = -1$ and $y = 2$

$\therefore 3xy\ (x - 5) + 4x = 3 \times (-1) \times 2\ (-1 - 5) + 4\ (-1) = (-6) \times (-6) + 4 \times (-1) = 36 - 4 = 32.$

(b) Given : $x = \frac{1}{4}$ and $y = \frac{-2}{3}$.

$$\therefore\ 3\ xy\ (x - 5) + 4\ x = \cancel{3} \times \frac{1}{4} \times \frac{-2}{\cancel{3}} \left(\frac{1}{4} - 5\right) + \cancel{4} \times \frac{1}{\cancel{4}}$$

$$= \frac{-2}{4}\left(\frac{1-20}{4}\right) + 1 = \frac{-1}{4} \times \frac{-19}{4} + 1 = \frac{19}{8} + \frac{8}{8} = \frac{27}{8} = 3\frac{3}{8}.$$

8. (a) $3\ x^2\ (x3 - y) - y^2\ (y^3 - x) = 3\ x^5 - 3\ x^2y - y^5 - y^2x$

(b) $-xy^2\ (x^2 + y^2 - 2\ xy + 4) = -x^3y^2 - xy^4 + 2\ x^2y^3 - 4\ xy^2$

(c) $-2a\ (x + 4) - 3\ a\ (2x - 3) + 6\ ax = -\ 2\ ax - 8\ a - 6\ ax + 9a + 6\ ax$

$= -2\ ax - \cancel{6\ ax} + \cancel{6\ ax} - 8a + 9a = -2\ ax + a = a - 2\ ax.$

(d) $(3x + 4y)\ (2x - y) - (x + 4y^2 - 6x^2) = 3x\ (2x - y) + 4y\ (2x - y) - x - 4\ y^2 + 6\ x^2$

$= 6\ x^2 - 3\ xy + 8\ xy - 4\ y^2 - x - 4y^2 + 6\ x^2$

$= 6\ x^2 + 6\ x^2 - 3\ xy + 8xy - 4\ y^2 - 4\ y^2 - x$

$= 12\ x^2 + 5\ xy - 8\ y^2 - x$

9. Given : a = 1, b = 2 and c = 0

(a) $5\,ab^2\,(a + b + c) = 5\,a^2b^2 + 5\,ab^3 + 5\,ab^2c$

$= 5 \times 1^2 \times 2^2 + 5 \times 1 \times 2^3 + 5 \times 1 \times 2^2 \times 0$

$= 20 + 40 + 0 = 60$ ...(i)

$= \text{Now } 5\,ab^2\,(a + b + c) = 5 \times 1 \times 2^2\,(1 + 2 + 0)$

$= 5 \times 4 \times 3 = 60$ ...(ii)

Both the results are same. Hence, verified.

(b) $2ab\,(a^2 - b^2 + abc) = 2\,a^3b - 2\,ab^3 + 2\,a^2b^2c$

$= a \times 1^3 \times 2 - 2 \times 1 \times 2^3 + 2 \times 1^2 \times 2^2 \times 0$

$= 4 - 2 \times 8 + 0 = 4 - 16 = -12$ ...(i)

Now $2\,ab\,(a^2 - b^2 + abc = 2 \times 1 \times 2\,(1^2 - 2^2 + 1 \times 2 \times 0)$

$= 4\,(1 - 4 + 0) = 4 \times (-3) = -12$ ...(ii)

From equations (i) and (ii), both the results are same. Hence, verified.

(c) $(a + b + c)^2 = (a + b + c)\,(a + b + c)$

$= a\,(a + b + c) + b\,(a + b + c) + c\,(a + b + c)$

$= a^2 + ab + ac + ab + b^2 + bc + ac + bc + c^2$

$= a^2 + b^2 + c^2 + 2\,ab + 2\,bc + 2\,ac$

$= 1^2 + 2^2 + 0^2 + 2 \times 1 \times 2 + 2 \times 2 \times 0 + 2 \times 1 \times 0$

$= 1 + 4 + 0 + 4 + 0 + 0 = 9.$ ...(i)

Now $(a + b + c)^2 = (1 + 2 + 0)^2 = 3^2 = 9$ ...(ii)

From equations (i) and (ii), both the results are same. Hence, verified.

(d) $(2a + 2b)\,(a + c) = a(2a + 2b) + c\,(2a + 2b)$

$= 2\,a^2 + 2\,ab + 2\,ac + 2\,bc$

$= 2 \times 1^2 + 2 \times 1 \times 2 + 2 \times 1 \times 0 + 2 \times 2 \times 0 = 2 + 4 = 6$ ...(i)

Now, $(2\,a + 2\,b)\,(a + c) = (2 \times 1 + 2 \times 2)\,(1 + 0) = (2 + 4) \times 1 = 6 \times 1 = 6$ ...(ii)

From equations (i) and (ii), both the results are same. Hence, verified.

10. $(a^3 - a^2 + a - 1) \times (2\,a^2 - a + 2)$

$= a^3\,(2\,a^2 - a + 2) - a^2\,(2\,a^2 - a + 2) - a\,(2\,a^2 - a + 2) - 1\,(2\,a^2 - a + 2)$

$= 2\,a^5 - a^4 + 2\,a^3 - 2\,a^4 + a^3 - 2\,a^2 + 2\,a^3 - a^2 + 2a - 2\,a^2 + a - 2$

$= 2\,a^5 - a^4 + 2\,a^4 - 2\,a^3 + a^3 - 2\,a^3 + 2\,a^2 - a^2 + 2\,a^2 - 2\,a + a - 2$

$= 2\,a^5 - 3\,a^4 + 5\,a^3 - 5\,a^2 + 3\,a - 2$

Exercise 6.3

1. (a) $\dfrac{x^{50}}{x^8} = x^{50-8} = x^{42}$

(b) $\dfrac{36\,\cancel{abc}}{6\,\cancel{ab}} = \dfrac{\cancel{36}^{\,6}\,c}{\cancel{6}} = 6\,c$

(c) $\dfrac{-9x^2y}{-3yx^2} = \dfrac{9\,\cancel{x^2}\,\cancel{y}}{3\,\cancel{x^2}\,\cancel{y}} = \dfrac{9}{3} = 3.$

(d) $\dfrac{15x\,\cancel{y^2}\,\cancel{z}}{5x^2\,\cancel{y^2}\,\cancel{z}} = \dfrac{15x}{5x^2} = \dfrac{3}{x}.$

2. (a) $10\,x^3y^3z \div 5\,x^3y^2z = \dfrac{10\,\cancel{x^3}\,\cancel{y^2}\,\cancel{z}}{5\,\cancel{x^3}\,\cancel{y^2}\,\cancel{z}} = \dfrac{10}{5} = 2.$

(b) $-25\,a^2bc \div 5\,ab = \dfrac{-25\,a^2\,\cancel{b}c}{5\,a\cancel{b}} = \dfrac{-25\,a^{2-1}c}{5} = \dfrac{-25\,ac}{5} = -5\,ac.$

(c) $-20\,a^2b^2c^2 \div -20\,abc^2 = \dfrac{\cancel{-20}\,a^2b^2\cancel{c}^2}{\cancel{-20}\,ab\cancel{c}^2} = a^{2-1} \times b^{2-1} = ab.$

(d) $14\,xyz^2 \div 10\,xy^2z^2 = \dfrac{14\,\cancel{x}\,y\,\cancel{z}^2}{10\,\cancel{x}\,y^2\,\cancel{z}^2} = \dfrac{14\,y}{10\,y^2} = \dfrac{7}{5\,y}.$

3. (a) $\dfrac{5a + 5b}{5} = \dfrac{\cancel{5}(a+b)}{\cancel{5}} = a + b.$

(b) $\dfrac{3x}{2} + 2x^2. = \dfrac{\cancel{8}^2\,(2x - y)}{\cancel{4}_1} = 2\,(2x - y) = 4x - 2y.$

(c) $\dfrac{36\,a^5 + 24\,a^4}{6\,a^3} = \dfrac{\cancel{6\,a}^3\,(6\,a^2 + 4\,a)}{\cancel{6\,a}^3} = 6\,a^2 + 4a.$

(d) $\dfrac{-18p - 9q}{9\,pq} = \dfrac{\cancel{9}\,(-2p - q)}{\cancel{9}\,pq} = \dfrac{-2p - q}{pq} = \dfrac{-2\cancel{p}}{\cancel{p}q} - \dfrac{\cancel{q}}{p\cancel{q}} = \dfrac{-2}{2} - \dfrac{1}{p}.$

4. (a) $\dfrac{2x^2 + x^2 + 4\,x^3}{2x} = \dfrac{3x^2 + 4\,x^3}{2x} = \dfrac{3x^2}{2x} + \dfrac{4\,x^3}{2x} = \dfrac{3\,x}{2} + 2x^2.$

(b) $\dfrac{6\,z^3 - 9\,z^2 + 3\,z}{3\,z} = \dfrac{\cancel{3\,z}\,\left(2\,z^2 - 3\,z + 1\right)}{\cancel{3\,z}} = 2\,z^2 - 3z + 1.$

(c) $\dfrac{x + 2x^3 - 3x^2}{-3x} = \dfrac{x}{-3x} + \dfrac{2x^3}{-3x} - \dfrac{3x^2}{-3x} = \dfrac{1}{-3} - \dfrac{2x^2}{3} + x$

(d) $\dfrac{8\,a^2b - 5\,ab + 10\,ab^2}{5\,ab} = \dfrac{8\,a^2b}{5\,ab} - \dfrac{5\,ab}{5\,ab} + \dfrac{10\,ab^2}{5\,ab} = \dfrac{8a}{5} - 1 + 2b.$

5. (a)

$p + 7 \;\big)\; \cancel{p^2} + 35 + 12\,p \;\big(\; p + 5$

$\quad \cancel{-p^2} \qquad + 7\,p$

$\quad + 35 + 5\,p$

$\quad 35 + 5\,p$

$\quad 0$

(b) $32\,x + 30 + 8x^2 = 8x^2 + 32x + 30$

$2x + 3 \;\big)\; \cancel{8x^2} + 32\,x + 30 \;\big(\; 4x + 10$

$\quad \cancel{-8x^2} + 12\,x$

$\quad 20x + 30$

$\quad 20x + 30$

$\quad 0$

(c)

$2m \;\; +2 \;\big)\; \cancel{6m^3} + 18m^2 - \;\; 12 \;\big(\; 3m^2 + 6m - \;\; 6$

$\quad \cancel{-6m^3} + \;\; 6m^2$

$\quad 12m^2 - \;\; 12$

$\quad - 12m^2 - 12m$

$\quad 12m - \;\; 12$

$\quad - \; 12m - \;\; 12$

$\quad 0$

(d) $3a - 1 \,\big)\, 3a^3 + 11a^2 + 5a - 3 \,\big(\, a^2 + 4a + 3$

$3a^3 - a^2$

$12a^2 + 5a - 3$

$12a^2 - 4a$

$9a - 3$

$9a - 3$

0

(e) $3z + 2 \,\big)\, 27z^3 - 64 \,\big(\, 9z^2 + 12z + 16$

$27z^3 - 36z^2$

$36z^2 - 64$

$36z^2 - 48z$

$48z - 64$

$48z - 64$

0

(f) $x - 1 \,\big)\, 5x^3 + x - 6 \,\big(\, 5x^2 + 5x + 6$

$5x^3 + 5x^2$

$5x^2 + x \; 6$

$- 5x^2 - 5x$

$6x - 6$

$6x - 6$

0

6. (a) $3x + 1 \,\big)\, 6x^3 + 20x^2 + x - 1 \,\big(\, 2x^2 + 6x - \frac{5}{3}$

$6x^3 + 2x^2$

$18x^2 + x - 1$

$18x^2 - 6x$

$-5x - 1$

$-5x - \frac{5}{3}$

$\frac{5}{3} - 1$

Thus, quotient = $2x^2 + 6x - \frac{5}{3}$ and reminder = $\frac{2}{3}$.

(b) $2x^3 + 7x + 5x^2 + 2 = 2x^3 + 5x^2 + 7x + 2$

$2x + 1 \,\big)\, 2x^3 + 5x^2 + 7x + 2 \,\big(\, x^2 + 2x - \frac{5}{2}$

$2x^3 + x^2$

$4x^2 + 7x + 2$

$4x^2 - 2x$

$5x - 2$

$-5x - \frac{5}{2}$

$\frac{5}{2} - 2 = \frac{-5 + 4}{2} = -\frac{1}{2}$

Thus, quotient = $x^2 + 2x + \frac{5}{2}$ and remainder = $-\frac{1}{2}$.

7. To get the answer, we will divide $2\,x^3 - 10\,x^2 + 14$ by $x - 4$.

$$\begin{array}{r|l|l}
x - 4 & \cancel{2x^3} - 10x^2 + 14 & 2x^2 + 2x - 8 \\
 & \underline{-\cancel{2x^3} + 8m^2} & \\
 & 2x^2 + 14 & \\
 & \underline{-\ 2x^2 - 8x} & \\
 & 8m + 14 & \\
 & \underline{-\ 12m + 32} & \\
 & -18 &
\end{array}$$

Here, the remainder is –18.
Thus, the number to be subtracted is –18.

8. To get the answer, we will first divide $5x^3 + 5x - 6x^2 - 3$ by $5\,x - 1$

$$\begin{array}{r|l|l}
5x - 1 & 5x^3 - 6x^2 + 5x - 3 & x^2 - x \\
 & \underline{-5x^3 + x^2} & \\
 & 5x^2 + 5x - 3 & \\
 & \underline{-\ 5x^2 + x} & \\
 & 4x - 3 &
\end{array}$$

Now, $5x - 1 - (4x - 3) = 5x - 1 - 4x + 3 = x + 2$.
Thus, required number (expression) to be added is $x + 2$.

9. $p + p^4 - p^3 + 2 - 3\,p^2 = p4 - p^3 - 3\,p^2 + p + 2$
and $2 + p^2 = 3p = p^2 - 3p + 2$.
Dividing we get,

$$\begin{array}{r|l|l}
p^2 - 3p + 21 & p^4 - p^3 + 3p^2 + p + 2 & p^2 + 2p + 1 \\
 & \underline{p^4 - 3p^3 + 2p^2} & \\
 & 2p^3 - 5p^2 + p + 2 & \\
 & \underline{-\ 2p^3 - 6p^2 + 4p} & \\
 & p^2 - 3p + 2 & \\
 & \underline{p^2 - 3p + 2} & \\
 & 0 &
\end{array}$$

As the reminder is 0, so $p^2 - 3\,p + 2$ is a factor of $p + p^4 - p^3 + 2 - 3p^2$.

10. $12a - 13a^2 + 4 - 3a^3 + a^4$
$a^4 - 3a^3 - 13a^2 + 12a + 4$

$$\begin{array}{r|l|l}
a^2 - 13p + 2 & a^4 - 3a^3 - 13a^2 + 12a + 4 & a^2 - 2a - 17 \\
 & \underline{a^4 - a^3 + 2a^2} & \\
 & 2a^3 - 15a^2 + 12a + 4 & \\
 & \underline{-\ 22^3 - 2p^2 - 4a} & \\
 & -\ 17a^2 + 16a + 4 & \\
 & \underline{-\ 17a^2 + 17p + 34} & \\
 & -a + 38 &
\end{array}$$

∴ Quotient = $a^2 - 2a - 17$ and remainder = $-a + 38$.
Now, dividend = divisor × quotient + remainder

RHS = divisor × quotient + remainder

$= (a^2 - a + 2)(a^2 - 2a - 17) + (-a + 38)$

$= a^2(a^2 - 2a - 17) - a(a^2 - 2a - 17) + 2(a^2 - 2a - 17) - a + 38$

$= a^4 - 2a^3 - 17a^2 - a^3 + 2a^2 + 17a + 2a^3 - 4a - 34 - a + 38$

$= a^4 - 2a^3 - a^3 + 17a^2 + 2a^2 + 2a^2 + 17a - 4a - a - 34 + 38$

$= a^2 - 3a^3 - 13a^2 + 12a + 4$

= dividend = LHS

Hence, Verified.

Exercise 6.4

1. (a) $(3a + 7)(3a + 7) = (3a + 7)^2$

Using the identity $(x + y)^2 = x^2 + 2xy + y^2$, we get

$(3a + 7)^2 = (3a)^2 + 2 \times 3a \times 7 + 7^2 = 9a^2 + 42a + 49.$

(b) $(9 - 6a)(9 + 6a) = 9^2 - (6a)^2 = 81 - 36a^2$ [Using the identity $x^2 - y^2 = (x - y)(x + y)$]

(c) $(2a^2 - 3b^2)(2a^2 - 3b^2) = (2a^2 - 3b^2)^2 = (2a^2)^2 - 2(2a^2)(3b^2) + (3b^2)^2 = 4a^4 - 12a^2b^2 + ab^4$

(d) $\left(\frac{1}{4}a^3 - \frac{1}{2}b\right)\left(\frac{1}{4}a^3 + \frac{1}{2}b\right) = \left(\frac{1}{4}a^3\right)^2 - \left(\frac{1}{2}b\right)^2$ [Using the identity $x^2 - y^2 = (x + a)(x - a)$]

$= \frac{1}{16}a^6 - \frac{1}{4}b^2$

(e) $(y + 5)(y + 8) = y^2 + (5 + 8)y + 40$ [Using $(x + a)(x + b) = x^2 + (a + b)x + ab$]

$= y^2 + 13y + 40$

(f) $(3x - 2)(3x - 5) = (3x)^2 + [(-2) + (-5)]3x + (-2)(-5) = 9x^2 - 21x + 10$

2. (a) We have $(5p + 2)^2$

Using the identity $(x + y)^2 = x^2 + 2xy + b^2$, we get

$(5p + 2)^2 = (5p)^2 + 2 \times 5p \times 2 + 2^2 = 25p^2 + 20p + 4$

(b) We have $(3a - 4)^2$.

Using the identity $(a - b)^2 = a^2 - 2ab + b^2$, we get

$(3a - 4)^2 = (3a)^2 - 2 \times 3a \times 4 + 4^2 = 9a^2 - 24a + 16$

(c) $(ab^2 - 3)^2 = (ab^2)^2 - 2 \times ab^2 \times 3 + 3^2 = a^2b^4 - 6ab^2 + 9.$

(d) Using the identity $(a + b)^2 = a^2 + b^2 + 2ab$, we get

$$\left(8 + \frac{1}{2}a^2\right)^2 = 8^2 + 2 \times 8 \times \frac{1}{2}a^2 + \left(\frac{1}{2}a^2\right)^2 = 64 + 8a^2 + \frac{1}{4}a^4$$

(e) Using the identity $(a - b)^2 = a^2 - 2ab + b^2$, we get

$(3m^2 - 2n^2)^2 = (3m^2)^2 - 2 \times 3m^2 \times 2n^2 + (2n^2)^2 = 9m^4 - 12m^2n^2 + 4n^4$

(f) Using the identity $(a - b)^2 = a^2 - 2ab + b^2$, we get

$(a^2 - b^2)^2 = (a^2)^2 - 2a^2b^2 + (b^2)^2 = a^4 - 2a^2b^2 + b^4$

(g) Using the identity $(a - b)^2 = a^2 - 2ab + b^2$, we get

$(a^3 - b^3)^2 = (a^3)^2 - 2a^3b^3 + (b^3)^2 = a^6 - 2a^3b^3 + b^6$

(h) $\left(x + \frac{1}{x}\right)^2 = x^2 + 2 \times \cancel{x} \times \frac{1}{\cancel{x}} + \frac{1}{x^2} = x^2 + 2 + \frac{1}{x^2}$

3. (a) $\left(8a+\frac{1}{12}b\right)^2=(8a)^2+2\times \cancel{8}^2 a+\frac{b}{\cancel{12}_3}+\left(\frac{b}{12}\right)^2=64a^2+\frac{4ab}{3}+\frac{b^2}{144}$

(b) $\left(2x-\frac{1}{6}\right)^2=(2x)^2-2x\times\frac{1}{6}+\left(\frac{1}{6}\right)^2=4x^2-\frac{2}{3}x+\frac{1}{36}$

(c) $\left(\frac{1}{4y}-x\right)^2=\left(\frac{1}{4y}\right)^2-2\times\frac{1}{4y}\times x+(x)^2=\frac{1}{16y^2}-\frac{x}{2y}+x^2$

(d) $\left(\frac{2}{3}a-\frac{1}{6}b\right)^2=\left(\frac{2}{3}a\right)^2-2\times\frac{2}{3}a\times\frac{1}{6}b+\left(\frac{1}{6}b\right)^2$

$=\frac{4}{9}a^2-\frac{4}{18}ab+\frac{1}{36}b^2=\frac{4}{9}a^2-\frac{2}{9}ab+\frac{1}{36}b.$

4. (a) $(99)^2=(100-1)^2=(100)^2-2\times 100\times 1+1^2=10000-200+1=9801.$

(b) $(104)^2=(100+4)^2=(100)^2+2\times 100\times 4+4^2=1000+800+16=10816.$

(c) $(91)^2=(90+1)^2=90^2+2\times 90\times 1+1^2=8100+180+1=8100+181=8281.$

(d) $95\times 94=(90+5)(90+4)=90^2+(5+4)\,90+5\times 4$ [$\because$ $(x+a)(x+b)=x^2+(a+b)x+ab$]

$=8100+9\times 90+20$

$=8100+810+20=8930.$

(e) $197\times 203=(200-3)(200+3)=200^2-3^2=40000-9=39991.$ [$a^2-b^2=(a+b)(a-b)$]

(f) $54\times 54-36\times 36=54^2-36^2=(54+36)(54-36)=90\times 18=1620.$ [$a^2-b^2=(a+b)(a-b)$]

(g) $8.4\times 8.4-8.3\times 8.3=(8.4)^2-(8.3)^2$

$=(8.4+8.3)(8.4-8.3)$ [$a^2-b^2=(a+b)(a-b)$]

$=16.7\times 0.1=1.67$

(h) $204\times 196=(200+4)(200-4)=(200)^2-4^2=40000-16=39984.$

5. (a) $6x=84^2-72^2=(84+72)(84-72)]=156\times 12=1872$

$\therefore x=1872\div 6=312$

(b) $12x=48^2-36^2=(48+36)(48-36)=84\times 12=1008.$

$\therefore x=1008\div 12=84$

6. (a) $(5.7)^2=(5+0.7)^2=5^2+2\times 5\times 0.7+(0.7)^2=25+10\times 0.7+0.49=32.49.$

(b) $8.5\times 8.5=(8+0.5)(8+0.5)=8^2+2\times 8\times 0.5+(0.5)^2=64+8+0.25=72.25.$

7. $25a^2+ab^2+30ab=(5a)^2+2\times 5a\times 3b+(3b)^2$

$=(5a+3b)^2=(5\times 6+3\times 8)^2=(30+24)^2=(54)^2=2916.$

8. (a) $x-\frac{1}{x}=4$ [Given]

$\therefore\left(x-\frac{1}{x}\right)^2=4^2$ [Squaring both sides]

$\Rightarrow x^2-2\times x\times\frac{1}{x}+\frac{1}{x^2}=16$

$\Rightarrow x^2+\frac{1}{x^2}-2=16$

$\Rightarrow\ x^2 + \frac{1}{x^2} = 16 + 2 = 18.$

(b) $x^2 + \frac{1}{x^2} = 18$ [From (a)]

$\Rightarrow\ \left(x^2 + \frac{1}{x^2}\right)^2 = 18^2$ [Squaring both sides]

$\Rightarrow\ x^4 + \frac{1}{x^4} + 2 \times x^2 \times \frac{1}{x^2} = 326$

$\Rightarrow\ x^4 + \frac{1}{x^4} + 2 = 326$

$\Rightarrow\ x^4 + \frac{1}{x^4} = 326 - 2 = 324$

$\therefore\ x^4 + \frac{1}{x^4} = 324.$

9. $x + y = 9$ [Given]

$\therefore\ (x + y)^2 = 9^2$ [Squaring both sides]

$\Rightarrow\ x^2 + y^2 + 2\,xy = 81$

$\Rightarrow\ x^2 + y^2 + 2 \times 16 = 81$ [Given xy = 16]

$\Rightarrow\ x^2 + y^2 = 81 - 32 = 49$

$\therefore\ x^2 + y^2 = 49$

10. $x - y = 10$ [Given]

$\therefore\ (x - y)^2 = 10^2$ [squaring both sides]

$\Rightarrow\ x^2 - 2 \times x \times y + y^2 = 100$

$\Rightarrow\ x^2 + y^2 - 2\,xy = 100$

$\Rightarrow\ x^2 + y^2 - 2 \times 9 = 100$ [Given xy = 9)

$\therefore\ x^2 + y^2 = 100 + 18 = 118.$

11. $\left(x - \frac{1}{x}\right)^2 = x^2 - 2 \times x \times \frac{1}{x} + \frac{1}{x^2} = x^2 + \frac{1}{x^2} - 2 = 18 - 2 = 16 = 4^2$ $\left[\text{Given } x^2 + \frac{1}{x^2} = 18\right]$

$\therefore\ x - \frac{1}{x} = 4.$

12. $(p - q)^2 = p^2 + q^2 - 2\,pq = 13 - \cancel{2} \times \frac{9}{\cancel{2}} = 13 - 9 = 4$

$(p - q)^2 = 2^2 \quad \therefore\ p - q = 2.$

13. $x - \frac{1}{x^2} = 27$

$\therefore \left(x^2 - \frac{1}{x^2}\right)^2 = 272$

$\Rightarrow\ x^4 + \frac{1}{x^4} - 2 \times x \times \frac{1}{x} = 729$

$\Rightarrow\ x^4 + \frac{1}{x^4} - 2 = 729$

$\Rightarrow\ x^4 + \frac{1}{x^4} = 729 + 2 = 731.$

14. $\left(z-\frac{1}{z}\right)^2 = z^2+\frac{1}{z^2}-2\times \cancel{z}\times\frac{1}{\cancel{z}} = 11-2=9$

$\Rightarrow \left(z-\frac{1}{z}\right)^2 = 3^2 \quad \Rightarrow \quad z-\frac{1}{z} = 3.$

15. (a) $16\,x^2+25\,y^2-\ 40\,xy = (4x)^2+(5y)^2-2\times 4x\times 5y = (4x-5y)^2$

$= [4\times 3-5\times(-1)]^2 = (12+5)^2 = 17^2 = 289$

(b) $\frac{9}{16}a^2-\frac{15a}{2}+25 = \left(\frac{3}{4}a\right)^2-2\times\frac{3}{4}a\times 5+5^2 = \left(\frac{3}{4}a-5\right)^2$

$= \left(\frac{3}{\cancel{4}_2}\times\cancel{2}^1-5\right)^2 = \left(\frac{3}{2}-\frac{10}{2}\right)^2 = \left(-\frac{7}{2}\right)^2 = \frac{49}{4} = 12\frac{1}{4}.$

Revision Exercise

1. See the **Answers** given in the book.

2. (a) $3\,p+p^{-1}+4$ is not a polynomial because the power of the veriable is negative.

(b) $2\,m^2-3m$ is a polynomial.

(c) $2\,x^2y-2\,x^2y-\sqrt[x]{y}+\sqrt{xy}$ is not a polynomial the power of variable is not a positive integer.

(d) $\frac{4}{5}\,a^3-5a+8$ is a polynomial.

3. See **Answers** given in the book.

4. (a) $6\,ab\times ab^3 = 6\,a^2b^4.$

(b) $-2xyz\times\frac{-3}{2}x^2y^2z^2 = \cancel{2}\times\frac{3}{\cancel{2}}x^3y^3z^3 = 3\,x^3y^3z^3.$

(c) $4\,pq^2\times\ -3\,p^2q^2 = 4\times(-3)\times p^3q^4 = -12\,p^3q^4.$

(d) $-\frac{3}{5}ab\times\left(-\frac{5}{6}a^2b^2c^2\right) = -\frac{3}{5}\times\left(-\frac{5}{6}\right)\times a^3b^3c^2 = \frac{1}{2}a^3b^3c^2.$

5. (a) $2a\,(3a+b) = 2a\times 3a+2a\times b = 6\,a^2+2\,ab$

(b) $-2\,xy\,(3x+y-\ 2\,xy) = -2\,xy\times 3x+(-2\,xy)\times y\ -\ (-2\,xy)\,(-2\,xy) = \ -6\,x^2y-2\,xy^2+4\,x^2y^2$

(c) $-5\,xy\,(-x^2y-8) = \ -5\,xy\times(-x^2y)-8\times(-5\,xy) = 5\,x^3y^2+40\,xy$

(d) $ab\,(2a+3b-ab) = ab\times 2a+ab\times 3b\ -\ ab\times ab = 2\,a^2b+3\,ab^2-\ a^2b^2.$

6. (a) $93p-q)\,(3p+q) = 3p\,(3p+q)-q\,(3p+q) = 9p^2+\cancel{3pq}-\ \cancel{3pq}-q^2 = 9\,p^2-q^2$

(b) $(9a-3)\,(3\,ab+1) = 9a\,(3\,ab+1)-3\,(3\,ab+1) = 27\,a^2b+9a\ -\ 9\,ab-3$

(c) $\left(\frac{2}{3}a^2-b^2\right)(3a^2+b) = \frac{2}{3}a^2\,(3\,a^2+b)-\ b^2\,(b)-b^2\times 3\,a^2$

$= \frac{2}{\cancel{3}}\times\cancel{3}\,a^4+\frac{2}{3}a^2b-3a^2b^2-b^3 = 2a^4+\frac{2}{3}a^2b-3a^2b^2-b^3.$

(d) $\left(\frac{4}{5}a^2+b^2\right)\left(a^2-\frac{1}{4}b\right) = \frac{4}{5}a^2\left(a^2-\frac{1}{4}b\right)+b^2\left(a^2-\frac{1}{4}b\right)$

$= \frac{4}{5}a^4-\frac{\cancel{4}}{5}\times\frac{1}{\cancel{4}}a^2b+a^2b^2-\frac{1}{4}b^3 = \frac{4}{5}a^4-\frac{1}{5}\,a^2b+a^2b^2-\frac{1}{4}b^3$

7. (a) $(73)^2 = (70 + 3)^2 = 70^2 + 2 \times 70 \times 3 + 3^2 = 4900 + 420 + 9 = 5329.$

(b) $(42)^2 = (40 + 2)^2 = 1600 + 2 \times 40 \times 2 + 4 = 1600 + 160 + 4 = 1764.$

(c) $(101)^2 = (100 + 1)^2 = 100^2 + 2 \times 100 \times 1 + 1^2 = 10000 + 200 + 1 = 10201.$

(d) Similar work to be done.

8. (a) $(x^2 - 2\,xy + y^2)\,(x - y) = (x - y)^2\,(x - y) = (x - y)^3 = (-1 - 2)^3 = (-3)^3 = 27.$ ...(i)

Now, $(x^2 - 2\,xy + y^2)\,(x - y)$

$= [(-1)^2 - 2 \times (-1) \times 2 + 2^2]\,(-1 - 2) = (1 + 4 + 4)\,(-3) = 9 \times (-3) = -27$ ...(ii)

From equations (i) and (ii) both the results are same.

Hence, verified.

(b) Similar work to be done.

9. $9\,m^2 - 12\,mn + 4n^2 = (3m)^2 - 2 \times 3m \times 2n + (2n)^2$

$$= (3m - 2n)^2 = \left(3 \times \frac{1}{2} + 2 \times \frac{-3}{4}\right)^2 = \left(\frac{3}{2} + \frac{6}{4}\right)^2 = \left(\frac{6 + 6}{4}\right)^2 = \left(\frac{\cancel{12}^{\,3}}{\cancel{4}_{\,1}}\right)^2 = 32 = 9.$$

10. (a)

$$\begin{array}{r|l|l} 2a - 3 & 6a^2 - 13a - 6 & 3a - 2 \\ & 6a^2 - 9a & \\ \hline & \quad -4a - 6 & \\ & \quad -4a - 6 & \\ \hline & \qquad 0 & \end{array}$$

(b) $$\frac{4x^2 - 12\,xy + 9y^2}{2x - 3y} = \frac{(2x)^2 - 2 \times 2x \times 3y + (3y)^2}{2x - 3y} = \frac{(2x - 3y)^2}{2x - 3y} = 2x - 3y.$$

11. (i) $x - \frac{1}{x} = 8.$

$\left(x - \frac{1}{x}\right)^2 = 8^2.$ [Squaring both sides]

$\Rightarrow\ x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x} = 64.$

(ii) $x^2 + \frac{1}{x^2} = 64 + 2 = 66.$ [From (i)]

$x^2 + \frac{1}{x^2} = 66$

$\therefore \left(x^2 + \frac{1}{x^2}\right)^2 = 66^2$

$\Rightarrow\ x^4 + \frac{1}{x^4} - 2 \times x \times \frac{1}{x} = 4356$

$\Rightarrow\ x^4 + \frac{1}{x^4} = 4356 - 2 = 4354.$

12. $p - q = 7$ [Given]

$\therefore\ (p - q)^2 = 7^2$ [Squaring both sides]

$\Rightarrow\ p^2 + q^2 - 2\,pq = 49$

$\Rightarrow\ p^2 + q^2 - 2 \times 9 = 49$

$\Rightarrow\ p^2 + q^2 = 49 + 18 = 67.$

Multiple Choice Questions

1. $36x^2 + 25y^2 - 60xy = (6x)^2 + (5y)^2 - 2 \times 6x \times 5y$

   $= (6x - 5y)^2 = [6 \times (-2) - 5 \times (-3)]^2 = (-12 + 15)^2 = 3^2 = 9.$

   Thus, the correct option is (b).

2. $$\begin{array}{r|llll|l} x - 3 & x^2 & - 6x & + 9 & & x - 3 \\ & x^2 & - 3x & & & \\ \hline & & 3x & + 9 & & \\ & & -\,3x & + 9 & & \\ \hline & & & 0 & & \end{array}$$

   $\therefore$ Quotient $= x - 3$

   Thus, the correct option is (a).

3. See the **Answers** given in the book.

4. $(3p + 1)(3p - 4) = 3p(3p - 4) + 1(3p - 4)$

   $= 9p^2 - 12p + 3p - 4$

   $= 9p^2 - 9p - 4.$

   Thus, the correct option is (b).

5. $\dfrac{42x^2y^2z}{-6xyz} = -7xy^2.$

   Thus, the correct option is (b).

6. Square of $-4m^2 = (-4m^2)^2 = 16m^4$. Thus, the correct option is (b).

7. $(-2a^2b^2)(0.5ab)(3a^3b^3) = (-a^3b^3)(3a^3b^3) = -3a^6b^6.$

   Thus, the correct option is (d).

8. $(x - y)^2 = x^2 - 2xy + y^2$

   Thus, the correct option is (d).

9. $\dfrac{x^2 - y^2}{x - y} = \dfrac{(x + y)(x - y)}{x - y} = x + y.$

   Thus, the correct option is (b).

10. $\dfrac{x^2 + 2xy + y^2}{x + y} = \dfrac{(x + y)^2}{x + y} = \dfrac{(\cancel{x + y})(x + y)}{\cancel{x + y}} = x + y.$

    Thus, the correct option is (b).

Mental Maths

A. See **Answers** given in the book.

B. See **Answers** given in the book.

Higher Order Thinking Skills (HOTS)

1. (i) $(489027)^2 - 489026 \times (489027 - 1) = (489027)^2 - 489026 \times 489027 + 489026$

   $= 489027(489027 - 489026) + 489026 = 489027 \times 1 + 489026 = 978053.$

(ii) $\dfrac{4.3 \times 4.3 + 3.7 \times 3.7 + 2 \times 4.3 \times 3.7}{4.3 + 3.7} = \dfrac{(4.3 + 3.7)^2}{4.3 + 3.7} = 4.3 + 3.7 = 8.$

2. Double of a – b = 2(a – b)
Triple of a + b = 3(a + b)
According to the question
$2(a - b) \times 3 (a + b) = 2 \times 3 (a - b) (a + b) = 6(a^2 - b^2) = 6 a^2 - 6 b^2$.

3. Speed $= \dfrac{\text{Distance covered}}{\text{Time taken}} = \dfrac{16x^2 + 48xy + 36y^2}{4} = 4x^2 + 12 xy + 9 y^2$

Thus, the speed of the train is $(4x^2 + 12 xy + 9y^2)$ km per hour.

7 Factorisation

Exercise 7.1

1. (a) $25 ab^2 = 5 \times 5 \times a \times b \times b$
(b) $15 x^2y^2 = 3 \times 5 \times x \times x \times y \times y$
(c) $12 xyz = 2 \times 2 \times 3 \times x \times y \times z$
(d) $18 p^2qr^2 = 2 \times 3 \times 3 \times p \times p \times q \times r \times r$

2. (a) $3 ab^2 = 3 \times a \times b \times b$
$6 ab = 2 \times 3 \times a \times b$
∴ Common factors are : 3, a and b
(b) $8p^2q = 2 \times 2 \times 2 \times p \times p \times q$
$4p = 2 \times 2 \times p$
∴ Common factors are : 2, 2 and p, i.e., 4 and p.
(c) $24 xyz = 2 \times 2 \times 2 \times 3 \times x \times y \times z$
$12 xy^2z = 2 \times 2 \times 3 \times x \times y \times y \times z$
∴ Common factors are : 2, 2, 3, x, y and z, i.e., 12, x, y and z.
(d) $10 mn = 2 \times 5 \times m \times n$
$5 m^3 = 5 \times m \times m \times m$
∴ Common factors are: 5 and m.

3. (a) $2p - 4 = 2 (p - 2)$
(b) $3 xy - 6 x^2y^2 = 3 xy (1 - 2 xy)$.
(c) $12a - 24 ab^2 = 12a (1 - 2 b^2)$.
(d) $24 a^3b - 36 a^2b^2c^2 = 12 ab (2 a^2 - 3 abc^2)$.
(e) $15x - 2 xy^2z^3 = 5x (3 - 4 y^2z^3)$.
(f) $m^2n - 5 m^2n^3 = m^2n (1 - 5 mn^3)$.
(g) $15 xyz - 5 x^2y + 10 xyz^3 = 5 xy (3z - x + 2 z^3)$
(h) $4 mn^2p - 8 mp^2 + m^2p^3n = mp (4 n^2 - 8p + mp^2n^2)$

4. (a) $x (y - 1) + 3 (y - 1) = (y - 1) (x + 3)$ [Taking y – 1 as common]
(b) $a(3x - 1) - b (3x - 1) = (3x - 1) (a - b)$ [Taking 3x – 1 as common]
(c) $2p (q - 1) - 7 (q - 1) = (q - 1) (2p - 7)$ [Taking q – 1 as common]
(d) $x (2m - n) + 6 (2m - n) = (2m - n) (x + 6)$ [Taking 2m – n as common]
(e) $x (6 - y) - (6 - y) = (6 - y) (x - 1)$ [Taking 6 – y as common]
(f) $a (a - 4) + 4 (4 - a) = a(a - 4) - 4 (a - 4) = (a - 4) (a - 4)$ [Taking a – 4 as common]

(g) $(a + b)(3a - 4) - (a + b)(2a - 3)$
$= (a + b)[(3a - 4) - (2a - 3)]$ [Taking a + b as common]
$= (a + b)[3a - 4 - 2a + 3] = (a + b)(a - 1)$

(h) $p(p - 2q) + r(p - 2q) + (2q - p)$
$= p(p - 2q) + r(p - 2q) - (p - 2q) = (p - 2q)(p - r - 1)$.

5. (a) $2b^2 + 4a^2 + a^2b^2 + 8 = 2b^2 + a^2b^2 + 4a^2 + 8$ [Arranging the terms]
$= b^2(2 + a^2) + 4(a^2 + 2)$
$= (b^2 + 4)(a^2 + 2)$

(b) $5a + 6ab + 3b + 10a^2$
$= 5a + 10a^2 + 3b + 6ab$ [Arranging the terms]
$= 5a(1 + 2a) + 3b(1 + 2a) = (5a + 3b)(1 + 2a)$

(c) $8xz + 4yz + 6xw + 3yw = 4z(2x + y) + 3w(2x + y) = (4z + 3w)(2x + y)$

(d) $6xy^2 + 2b^2y2 + 3x + b^2$
$= 6xy^2 + 3x + 2b^2y^2 + b^2$ [Arranging the terms]
$= 3x(2y^2 + 1) + b^2(2y^2 + 1) = (3x + b^2)(2y^2 + 1)$

(e) $xyz - xy + 1 - z = xyz - xy - z + 1 = xy(z - 1) - 1(z - 1) = (xy - 1)(z - 1)$

(f) $m^2n - mr^2 - mn + r^2 = m^2n - mn - mr^2 - r^2 = mn(m - 1) - r^2(m - 1) = (mn - r^2)(m - 1)$

Exercise 7.2

1. $y^2 + 2y - 15 = y^2 + 5y - 3y - 15 = y(y + 5) -3(y + 5) = (y + 5)(y - 3)$
2. $x^2 + x - 12 = x^2 + 4x - 3x - 12 = x(x + 4) -3(x + 4) = (x - 3)(x + 4)$
3. $m^2 - 6m + 8 = m^2 - 4m - 2m + 8 = m(m - n) -2(m - n) = (m - 2)(m - 4)$
4. $7x^2 - 19x - 6 = 7x^2 - 21x + 2x - 6 = 7x(x - 3) + 2(x - 3) = (7x + 2)(x - 3)$
5. $3x^2 - 10x + 8 = 3x^2 - 6x - 4x + 8 = 3x(x - 2) -4(x - 2) = (3x - 4)(x - 2)$
6. $x^2 - 2x - 15 = x^2 - 5x + 3x - 15 = x(x - 5) + 3(x - 5) = (x + 3)(x - 5)$
7. $x^2 + 11xy + 18y^2 = x^2 + 9xy + 2xy + 18y^2 = x(x + 9y) + 2y(x + 9y) = (x + 2y)(x + 9y)$
8. $m^2 - 10m + 21 = m^2 - 7m - 3m + 21 = m(m - 7) -3(m - 7) = (m - 3)(m - 7)$
9. $3p^2 - 12pqr - 15q^2r^2 = 3p^2 - 15pqr + 3pqr - 15q^2r^2$
$= 3p(p - 5qr) + 3qr(p - 5qr) = (3p + 3qr)(p - 5qr)$
10. $x^2 - 7x - 144 = x^2 - 16x + 9x - 144 = x(x - 16) + 9(x - 16) = (x + 9)(x - 16)$
11. $11x^2 + 54x + 63 = 11x^2 + 33x + 21x + 63 = 11x(x + 3) + 21(x + 3) = (11x + 21)(x + 3)$
12. $z^2 + 5yz - 24y^2 = z^2 + 8yz - 3yz - 24y^2 = z(z + 8y) -3y(z + 8y) = (z - 3y)(z + 8y)$
13. $p^2 - 8p - 65 = p^2 - 13p + 5p - 65 = p(p - 13) + 5(p - 13) = (p + 5)(p - 13)$
14. $15a^2 - 26a + 8 = 15a^2 - 20a - 6a + 8 = 5a(3a - 4) - 2(3a - 4) = (5a - 2)(3a - 4)$
15. $28 - 31m - 5m^2 = -5m^2 - 31m + 28 = -5m^2 - 35m + 4m + 28$
$= -5m(m + 7) + 4(m + 7) = (-5m + 4)(m + 7)$
16. $p^2 - 6p - 40 = p^2 - 10p + 4p - 40 = p(p - 10) + 4(4 - 10) = (p + 4)(p - 10)$

Revision Exercise

1. See the **Answers** given the book.

2. (a) $9a^2 = 3 \times 3 \times a \times a$

$12a = 3 \times 4 \times a$

Common factors are 3 and a.

$\therefore$ HCF of $9a^2$ and $12a$ is $3 \times a = 3a$.

(b) $15p^2q^2 = 3 \times 5 \times p \times p \times q \times q$

$24pq = 2 \times 2 \times 2 \times 2 \times 3 \times p \times q$

Common factors are 3, p and q.

$\therefore$ HCF of $15p^2q^2$ and $24pq$ is

$3 \times p \times q = 3pq$.

(c) $12a^2b^3 = 2 \times 2 \times 3 \times a \times a \times b \times b \times b$

$15ab = 3 \times 5 \times a \times b$

$98a^2b^2 = 2 \times 7 \times 7 \times a \times a \times b \times b$

Common factors are a and b.

$\therefore$ HCF of $12a^2b^3$, $15ab$ and $98a^2b^2$ is $a \times b = ab$.

(d) $36xyz^2 = 2 \times 2 \times 3 \times 3 \times x \times y \times z \times z$

$72x^2z^2 = 2 \times 2 \times 2 \times 3 \times 3 \times x \times x \times z \times z$

Common factors are : 2, 2, 3, 3, x, z and z.

$\therefore$ HCF of $36xyz^2$ and $72x^2z^2$ is $2 \times 2 \times 3 \times 3 \times x \times z \times z = 36xz^2$.

2. (a) $3x - 12 = 3(x - 4)$. (b) $6a - 6b = 6(a - b)$.

(c) $4pq - 2p + 8p^2 = 2p(2q - 1 + 4p)$ (d) $8xy - 4xy^2z = 4xy(2 - yz)$.

4. (a) $24x^{12}y^6 - 36x^6y^6 = 12x^6y^6(2x^6 - 3)$. (b) $25p - 15q = 5(5p - 3q)$.

(c) $45ab - 60ab^2 = 15ab(3 - 4b)$ (d) $80xy - 40y^2z^2 = 40y(2x - yz^2)$

5. (a) $pq - pq - ps = p(q - r - s)$ [Taking p as common]

(b) $x^2y4 - x^4y^2 - x^4y^4 = x^2y^2(1 - x^2 - x^2y^2)$ [Taking x^2y^2 as common]

(c) $6x(2x - y) + 7y(2x - y) = 2x - y(6x + 7y)$ [Taking $2x - y$ as common]

(d) $bcpq + apq - bcr - ar = pq(bc + a) - r(bc + a) = (pq - r)(bc + a)$

6. (a) $y^4 + 49 + 14y^2 = (y^2)^2 + (7)^2 + 2 \times 7 \times y^2 = (y^2 + 7)^2$

(b) $16a^2b - \frac{b}{16x^2} = b\left(16a^2 - \frac{1}{16x^2}\right)$ [Taking b as common]

$= b\left[(4a)^2 - \left(\frac{1}{4x}\right)^2\right] = b\left(4a + \frac{1}{4x}\right)\left(4a - \frac{1}{4x}\right)$ [Using identity $a^2 - b^2 = (a - b)(a + b)$]

(c) $100(x + y)^2 - 81(a + b)^2 = 10^2(x + y)^2 - 9^2(a + b)^2$

$= [10(x + y)]^2 - [a(a + b)]^2 = [10(x + y) + 9(a + b)][10(x + y) - 9(a + b)]$

(d) $16p^4 - q^4 = (4p^2)^2 - (q^2)^2 = (4p^2 - q^2)(4p^2 - q^2)$ [Using identity $a^2 - b^2 = (a + a)(a - b)$

(e) $a^3 - 144a = 1(a^2 - 144)$ [Taking a as common]

$= a[(a^2) - (12)^2] = a(a + 12)(a - 12)$

[Using identity $a^2 - b^2 = (a + b)(a - b)$]

(f) $64 - (x + 1)^2 = 8^2 - (x + 1)^2 = (8 - x - 1)(8 - x + 1) = (7 - x)(9 + x)$

7. (a) $6x^2 - 13xy + 2y^2 = 6x^2 - 12xy - xy + 2y^2 = 6x(x - 2y) - y(x - 2y) = (6x - y)(x - 2y)$

(b) $2m^2 + 44 - 19m = 2m^2 - 19m + 44 = 2m^2 - 11m - 8m + 44$

$= m(2m - 11) - 4(2m - 11) = (m - n)(2m - 11)$

(c) $36a^2 + 12abc - 15b^2c^2 = 3(12a^2 + 4abc - 5b^2c^2)$

$= 3(12a^2 + 10abc - 6abc - 5b^2c^2)$

$= 3[2a(6a + 5bc) - bc(ba - 5bc)] = 3[(2a - b)(6a + 5bc)]$

(d) $2p^2 + 16p + 24 = 2p^2 + 12p + 4p + 24 = 2p(p + 6) + 4(p + 6) = (2p + 4)(p + 6)$

(e) $9 + 36x^2 - 36x = 36x^2 - 36x + 9 = 36x^2 - 18x - 18x + 9$
$= 18x(2x - 1) - 9(2x - 1) = (18x - 9)(2x - 1)$

(f) $3z^2 + z^4 - 4 = z^4 + 3z^2 - 4 = z^4 + 4z^2 - z^2 - 4 = z^2(z^2 + 4) - 1(z^2 - 4) = (z^2 - 1)(z^2 + 4)$

(g) $a^4 - a^2 - 20 = a^4 - 5a^2 + 4a^2 - 20 = a^2(a^2 - 5) + 4(a^2 - 5) = (a^2 + 4)(a^2 - 5)$

(h) $x^2 - 10x + 21 = x^2 - 7x - 3x + 21 = x(x - 7) - 3(x - 7) = (x - 3)(x - 7)$

8.

$$
\begin{array}{r|rrr|l}
x - 3 & x^2 & -7x & -30 & x + 10 \\
 & x^2 & -3x & & \\
\hline
 & & 10x & -30 & \\
 & & -10x & -30 & \\
\hline
 & & & 0 &
\end{array}
$$

As the remainder is 0, so $x - 3$ is a factor of $x^2 - 7x - 30$.

9. To find the other factor, we will divide $a^2 - 8a - 65$ by the given factor.

$$
\begin{array}{r|rrr|l}
a + 5 & a^2 & -8a & -65 & a - 13 \\
 & a^2 & +5a & & \\
\hline
 & & -13a & -65 & \\
 & & -13a & -65 & \\
\hline
 & & & 0 &
\end{array}
$$

Hence, the other factor is $a - 13$.

10. The required algebraic expression is the product of both the given factors.
$\therefore (2p + 1)(p - 18) = 2p(p - 18) + 1(p - 18) = 2p^2 - 36p + p - 18 = 2p^2 - 35p - 18.$
Hence, the required algebraic expression is $2p^2 - 35p - 18$.

11. $\dfrac{(6.38)^2 - (0.38)^2}{6} = \dfrac{(6.38 + 0.38)(6.38 - 0.38)}{6}$ [Using the identity $a^2 - b^2 = (a - b)(a + b)$]
$= \dfrac{6.76 \times \cancel{6}}{\cancel{6}} = 6.76.$

12. $(3x - 2y) = 9$
$\Rightarrow (3x - 2y)^2 = 9^2$ [Squaring both sides]
$\Rightarrow 9x^2 + 4y^2 - 2 \times 3x \times 2y = 81$
$\Rightarrow 9x^2 + y^2 - 12xy = 81$
$\Rightarrow 9x^2 + y^2 - 12 \times 7 = 81$
$\Rightarrow 9x^2 + y^2 = 81 + 84 = 165.$

13. (i) $a - \dfrac{1}{a} = 10$ [Given]

$\Rightarrow \left(a - \dfrac{1}{a}\right)^2 = 10^2$ [Squaring both sides]

$\Rightarrow a^2 + \dfrac{1}{a^2} - 2 \times \cancel{a} \times \dfrac{1}{\cancel{a}} = 100$

$\Rightarrow a^2 + \dfrac{1}{a^2} - 2 = 100$

$\Rightarrow a^2 + \dfrac{1}{a^2} = 100 + 2 = 102$

(ii) $a^2 + \frac{1}{a^2} = 102$ [From (i)]

$\Rightarrow \left(a^2 + \frac{1}{a^2}\right)^2 = (102)^2$ [Squaring both sides]

$\Rightarrow a^4 + \frac{1}{a^4} + 2 \times \frac{1}{\cancel{a^2}} \times \cancel{a^2} = 10404$

$\Rightarrow a^4 + \frac{1}{a^4} = 10404 - 2 = 10402$

Multiple Choice Questions

1. ab + a + b + 1 = a (b + 1) + 1 (b + 1) = (a + 1) (b + 1) Thus, the correct option is (b).

2. See the **Answers** given in the book.

3. 8(x + y) + 4a (x + y) = (8 + 4a) (x + y) = 4 (2 + a) (x + y)
   Thus, the correct option is (d).

4. $16\,x^3y^2 = 2 \times 2 \times 2 \times 2 \times x \times x \times x \times y \times y$
   $12\,x^2y = 2 \times 2 \times 3 \times x \times x \times y$
   Common factors are 2, 2, x, x and y.
   $\therefore$ HCF $= 2 \times 2 \times x \times x \times y = 4\,x^2y$
   Thus, the correct option is (c).

5. $x^2 + y^2 - z^2 - 2\,xy = x^2 + y^2 - 2\,xy - z^2 = (x - y)^2 - z^2 = (x - y - z)\,(x - y + z)$
   Thus, the correct option is (a).

6. $(4a - 1)\,(a + b + 1) = 4a\,(a + b + 1)\ -1\,(a + b + 1)$
   $= 4\,a^2 + 4\,ab + 4\,a - a - b - 1 = 4\,a^2 + 4\,ab + 3\,a - b - 1$
   Thus, the correct option is (d).

7. Let the numbers be x and y such that x > y.
   Then xy = 12
   Also x + y = 8 $\therefore$ x = 8 – y
   Now 8 – y (y) = 12
   $\Rightarrow 8 - y^2 = 12$
   $\Rightarrow -y^2 = 12 - 8 = 4$
   $\Rightarrow y - 2$
   $\therefore$ xy = 12
   $\Rightarrow x = \frac{12}{2} = 6.$
   Thus, the correct option is (b).

8. $x^2 + \frac{1}{x^2} + 2 \times x + \frac{1}{x} = \left(x + \frac{1}{x}\right)^2$. Thus, the correct option is (b).

9. $(x - 10)^2 + (10 - x) = (x - 10)\,(x - 10) + (10 - x) = (x - 10)\,(x - 11)$
   Thus, the correct option is (b).

10. $x^2 - (x + y)2 = [x - (x + y)]\,[x + (x + y)] = (\cancel{x} - \cancel{x} - y)\,(x + x + y) = -y\,(2\,x + y)$
    Thus, the correct option is (c).

Mental Maths

A. See the **Answers** given in the book.

B. See the **Answers** given in the book.

Higher Order Thinking Skills (HOTS)

1. $(2x + 3y)^2 - 5(2x + 3y) - 14$

$= (2x + 3y)^2 - 7(2x + 3y) + 2(2x + 3y) - 14$

$= (2x + 3y)(2x + 3y - 7) + 2(2x + 3y - 7)$

$= (2x + 3y + 2)(2x + 3y - 7)$

2. $$\frac{(p-1)(p-2)(p^2+14-9p)}{(p-7)(p^2+2-3p)} = \frac{\cancel{(p^2-3p+2)}(p^2-7p-2p+14)}{(p-7)\cancel{(p^2+2-3p)}}$$

$$= \frac{p(p-7)-2(p-7)}{p-7} = \frac{(p-2)\cancel{(p-7)}}{\cancel{p-7}} = p - 2.$$

3. $(a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$

LHS $= (a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = \cancel{a^2} - \cancel{b^2} + \cancel{b^2} - \cancel{c^2} + \cancel{c^2} - \cancel{a^2} = 0 =$ RHS.

Hence, proved.

8 Linear Equations in One Variable

Exercise 8.1

1. (a) $2x + 5 = 3x + 1$

$\Rightarrow 2x - 3x = 1 - 5$ [By transposition]

$\Rightarrow -x = -4$

$\therefore x = 4$

(b) $\frac{6x}{5} = x - 3$

$\Rightarrow 6x = 5x - 15$

$\Rightarrow 6x - 5x = -15$ [By transposition]

$\therefore x = -15.$

(c) $5m + 2m + 4 = 3m - 11 + 8m$

$\Rightarrow 7m + 4 = 11m - 11$ [Adding similar terms]

$\Rightarrow 7m - 11m = -11 - 4$ [By transposition]

$\Rightarrow -4m = -15$

$\therefore m = \frac{15}{4}.$

(d) $0.4(x + 3) - 0.2(x - 1) = 3x + 3$

$\Rightarrow 0.4x + 1.2 - 0.2x + 0.2 = 3x + 3$

$\Rightarrow 0.2x + 1.4 = 3x + 3$

$\Rightarrow 0.2x - 3x = 3 - 1.4$

(e) $4p - 21 = \frac{-p}{5}$

$\Rightarrow 20p - 105 = -p$

$\Rightarrow 20p + p = 105$

$\Rightarrow 21p = 105$

$\therefore p = 105 \div 21 = 5.$

(f) $0.09\ a + 0.45\ a = 8 - 0.26\ a$

$\Rightarrow 0.54\ a + 0.26\ a = 8$

$\Rightarrow 0.80\ a = 8$

$\Rightarrow a = \frac{8}{0.8} = \frac{80}{8} = 10.$

$\therefore a = 10.$

2. (a) $\frac{2p}{5} = \frac{p+1}{3}$

$\Rightarrow 3 \times 2p = 5\ (p + 1)$ [By cross multiplication]

$\Rightarrow 6p = 5p + 5$

$\Rightarrow 6p - 5p = 5$ [By transposition]

$\therefore p = 5.$

(b) $\frac{7x - 5}{4x} = 3.$

$\Rightarrow 7x - 5 = 4x \times 3$ [By cross multiplication]

$\Rightarrow 7x - 5 = 12\ x$

$\Rightarrow 7x - 12\ x = 5$ [By Transposition]

$\Rightarrow -5\ x = 5$

$\therefore x = \frac{5}{-5} = -1.$

(c) $\frac{3+2m}{5} = \frac{2m + 5}{8}$

$\Rightarrow 8\ (3 + 2m) = 5\ (2m + 5)$ [By cross multiplication]

$\Rightarrow 24 + 16\ m = 10\ m + 25$

$\Rightarrow 16\ m - 10\ m = 25 - 24$ [By transposition]

$\Rightarrow 6m = 1$

$\therefore m = \frac{1}{6}.$

(d) $\frac{2x - 5}{3x - 1} = \frac{3}{4}$

$\Rightarrow 8x - 20 = 9x - 3$ [By Cross multiplication]

$\Rightarrow 8x - 9x = -3 + 20$ [By transposition]

$\Rightarrow -x = 17$

$\therefore x = -17.$

(e) $\frac{(3 + 2x) - (5x - 6)}{3} = \frac{-6x - 24}{3} + \frac{27}{3} = \frac{(3 + 2x) - (5x - 6)}{3} = \frac{-6x - 24 + 27}{3}$

$\Rightarrow$ $(3 + 2x) - (5x - 6) = -6x + 3$ [Cancelling 3]

$\Rightarrow$ $3 + 2x - 5x + 6 = -6x + 3$

$\Rightarrow$ $-3x + 6x = 3 - 9$

$\Rightarrow$ $3x = -6$

$\Rightarrow$ $x = -6 \div 3 = -2$

$\therefore$ $x = -2$

(f) $\dfrac{1}{1+2z} = \dfrac{1}{3z-1}$

$\Rightarrow$ $3z - 1 = 1 + 2z$

$\Rightarrow$ $3z - 2z = 1 + 1$

$\Rightarrow$ $z + 2$

$\therefore$ $z = 2.$

(g) $\dfrac{x-2}{2} - \dfrac{6x-1}{6} = 0$

$\Rightarrow$ $6(x - 2) - 2(6x - 1) = 0$

$\Rightarrow$ $6x - 12 - 12x + 2 = 0$

$\Rightarrow$ $-6x - 12 + 2 = 0$

$\Rightarrow$ $-6x = 10$

$\Rightarrow$ $x = \dfrac{10}{-6} = \dfrac{-5}{3}.$

$\therefore$ $x = \dfrac{-5}{3}.$

(h) $\dfrac{a^2 - (a+2)(a+1)}{5a+1} = 5.$

$\Rightarrow$ $a^2 - (a^2 + 3a + 2) = 5(5a + 1)$

$\Rightarrow$ $a^2 - a^2 - 3a - 2 = 25a + 5$

$\Rightarrow$ $-3a - 25a = 5 + 2$

$\Rightarrow$ $-28a = 7$

$\Rightarrow$ $a = \dfrac{7}{-28} = \dfrac{1}{-4}$

$\therefore$ $a = \dfrac{1}{-4}.$

Exercise 8.2

1. Let the smaller number be x. Then the greater number = 6x.
According to the question,
$6x - 8 = 5x$
$\Rightarrow$ $6x - 5x = 8$
$\Rightarrow$ $x = 8$
Thus, the smaller number is 8 and the greater number is $8 \times 6 = 48$.

2. Let the numbers be 5x and 7x. Then according to the question,
$5x + 7x = 108.$
$\Rightarrow$ $12x = 108$

$\Rightarrow$ $x = 108 \div 12 = 9$

$\therefore$ $5x = 5 \times 9 = 45$ and $7x = 7 \times 9 = 63$

Hence, the numbers are 45 and 63.

3. Let the number be x. Then its one fourth = $\frac{x}{4}$ and it one eighth = $\frac{x}{8}$

According to the question,

$\frac{x}{4} - 10 = \frac{x}{8}$

$\Rightarrow$ $\frac{x}{4} - \frac{x}{8} = 10$

$\Rightarrow$ $\frac{2x - x}{8} = 10$

$\Rightarrow$ $x = 10 \times 8 = 80.$

Hence, the number is 80.

4. Let the first part x. Then the second part = 2x + 10, and third number = x + 75

According to the question,

$x + 2x + 10 + x + 75 = 585$

$\Rightarrow$ $4x + 85 = 585$

$\Rightarrow$ $4x = 585 - 85 = 500$

$\Rightarrow$ $x = 500 \div 4 = 125.$

$\therefore$ First part = 125, Second part = 2 x + 19 = 250 + 10 = 260

and third part = x + 75 = 125 + 75 = 200.

5. Let the three consecutive numbers be x, x + 1 and x + 2.

Then, $x + x + 1 + x + 2 = 708$

$\Rightarrow$ $3x + 3 = 708$

$\Rightarrow$ $3x = 708 - 3 = 705$

$\Rightarrow$ $x = 705 \div 3 = 235.$

Thus, the required consecutive numbers are 235, 236 and 237.

6. Let the required number be x, x + 2 and x + 4.

Then according to the question,

$\Rightarrow$ $x + x + 2 + x + 4 = 462.$

$\Rightarrow$ $3x + 6 = 462$

$\Rightarrow$ $3x = 462 - 6 = 456$

$\Rightarrow$ $x = \frac{456}{3} = 152.$

Thus, the required numbers are 152, 154 and 156.

7. Let the required numbers be x, x + 2 and x + 4.

Then according to the question,

$\Rightarrow$ $x + x + 2 + x + 4 = 267$

$\Rightarrow$ $3x + 6 = 267$

$\Rightarrow$ $3x = 267 - 6 = 261$

$\Rightarrow$ $x = \frac{261}{3} = 87$

Thus, the required number are 87, 89 and 91.

8. Let the three consecutive multiples of 11 be x, x + 11 and x + 22.
Then x + x + 11 + x + 22 = 297
$\Rightarrow$ 3x + 33 = 297
$\Rightarrow$ 3x = 297 − 33 = 264
$\Rightarrow$ $x = \frac{264}{3} = 88$
Thus, the required multiples of 11 and 88, 99 and 110.

9. Let the length of the rectangular garden be x m. Then its breadth will be $\frac{x}{2} - 5$.
Perimeter of the garden = 260 m [Given]
$\Rightarrow$ 2 (length + breadth) = 260 m
$\Rightarrow$ length + breadth = 260 m ÷ 2 = 130 m
$\Rightarrow$ $x + \frac{x}{2} - 5 = 130$ m
$\Rightarrow$ 2x + x − 10 = 130 × 2m = 260 m
$\Rightarrow$ 3x = 260 + 10 = 270 m
$\Rightarrow$ $x = \frac{270}{3} = 90$ m
$\therefore \frac{x}{2} - 5 = \frac{90}{2} - 5 = 45 - 5 = 40$m.
Thus, the length and the breadth of the rectangular garden are respectively 90 m and 40 m.

10. Let the present age of Neha be x years. Then her mother's present age will be (x + 20) years.
Then years ago:
Neha's age = (x − 10) years
Her mother's age = (x + 20 − 10) = (x + 10) years.
According to the question,
3(x − 10) = x + 10
$\Rightarrow$ 3x − 30 = x + 10
$\Rightarrow$ 3x − x = 10 + 30
$\Rightarrow$ 2x = 40
$\Rightarrow$ x = 40 ÷ 2 = 20
Thus, Neha's persent age is 20 years and that of her mother is 20 + 20 = 40 years.

11. Let Sunil's present age be x years. Then Mr. Mehta's present age will be 6 years.
After six years:
Sunil's age will be (x + 6) years.
Mr. Mehta's age will be (6x + 6) years
According to the questions
3(x + 6) = 6x + 6
$\Rightarrow$ 3x + 18 = 6x + 6
$\Rightarrow$ 3x − 6x = 6 − 18
−3x = −12
$\Rightarrow$ x = 12 ÷ 3 = 4
Thus, the present age of Mr. Mehta is 6 × 4 = 24 years

12. Let the present age of Ananya be x years. Then the present age of her mother will be (x + 20) years.

In 4 years :

Age of Ananya will be (x + 4) years

Age of her mother will be (x + 20 + 4) years

According to the question,

$3 (x + 4) = x + 24$

$\Rightarrow = 3x + 12 = x + 24$

$\Rightarrow 3x - x = 24 - 12$

$\Rightarrow 2x = 12$

$\Rightarrow x = 12 \div 2 = 6.$

Thus, the present ages of Ananya and her mother are 6 years and 24 years respectively.

13. Let the ones place digit be x. Then the tens place digit will be 4 x.

The number formed = $10 \times 4x + x = 41x$

On reversing the digits :

Ones place digit = 4x and tens place digit = x.

New number formed = $10x + 4x = 14x$.

According to the question,

$41x - 14x = 54$

$\Rightarrow 27x = 54$

$\Rightarrow x = 2$

$\therefore$ One place digit = 2 and tens place digit = 8.

Thus, the number is 82.

14. Let the number be x. Then 82 less than x = x – 82.

Difference = 112 – x

According to the question,

x – p2 : 112 – x = 1 : 1

$$\Rightarrow \frac{x - 82}{112 - x} = \frac{1}{1}$$

$\Rightarrow x - 82 = 112 - x$

$\Rightarrow x + x = 112 + 82$

$\Rightarrow 2x = 194$

$\Rightarrow x = 97$

Thus, the required number is 97.

15. Let the speed of the steamer be x km/h.

Speed of water (stream) = 1.5 km/h

Speed of the steamer going down stream = (x + 1.5) km/h

Speed of the steamer going up stream = (x – 1.5) km/h.

$\therefore$ Distance covered = speed × time

Distance covered by the streamer down stream = Distance cover by it upstream.

$$\Rightarrow (x + 1.5)\,4 = (x - 1.5) \times 5\frac{1}{2}$$

$\Rightarrow$ $4x + 6 = \dfrac{5x}{2} - 8.25$

$\Rightarrow$ $4x - \dfrac{5x}{2} = -8.25 - 6$

$\Rightarrow$ $\dfrac{8x - 5x}{2} = -14.25$

$\Rightarrow$ $3x = 28.5$

$\Rightarrow$ $x = 28.5 \div 3 = 9.7.$

Hence the speed of the steamer in still water in 9.7 km/h.

Revision Exercise

1. (a) $p - 7 = 8 - 5p$

$\Rightarrow$ $3p + 5p = 8 + 7$

$\Rightarrow$ $8p = 15$

$\Rightarrow$ $p = \dfrac{15}{8}.$

Verification :

LHS $= 3 \times \dfrac{15}{8} - 7 = \dfrac{45 - 56}{8} = \dfrac{-11}{8}.$ RHS $= 8 - 5p = 8 - \dfrac{5 \times 15}{8} = \dfrac{64 - 75}{8} = \dfrac{-11}{8}.$

As LHS = RHS, hence verified.

(b) $\dfrac{3m}{5} = m - 4$

$\Rightarrow$ $\dfrac{3m}{5} - m = -4$

$\Rightarrow$ $3m - 5m = -20$

$\Rightarrow$ $-2m = -20$

$\Rightarrow$ $m = 10$

Verification :

LHS $= \dfrac{3m}{5} = \dfrac{3 \times 10}{5} = 6$ RHS $= m - n = 10 - 4 = 6$

As LHS = RHS, hence verified

(c) Similar work to be done.

(d) $3(x - 2) + 3(x - 3) = 4(x - 1)$

$\Rightarrow$ $3x - 6 + 2x - 6 = 4x - 4$

$\Rightarrow$ $5x - 12 = 4x - 4$

$\Rightarrow$ $5x - 4x = -4 + 12$ $\Rightarrow$ $x = 8$

Similar work to be done for verification as (a) and (b)

(e) $0.04(6x - 1) = 5 + 0.2x$

$\Rightarrow$ $0.24x - 0.04 = 5 + 0.2x$

$\Rightarrow$ $0.24x - 0.2x = 5 + 0.04$

$\Rightarrow$ $0.04x = 5.04$

$\Rightarrow$ $x = \dfrac{5.04}{0.04} = 126.$

Similar work to be done for verification.

(f) $0.5x + \frac{x}{3} = 7 + 0.25\,x$

$\Rightarrow$ $1.5\,x + x = 21 + 0.75\,x$

$\Rightarrow$ $1.5\,x - 0.75\,x + x = 21$

$\Rightarrow$ $0.75\,x + x = 21$

$\Rightarrow$ $1.75\,x = 21$

$\Rightarrow$ $x = \frac{21}{1.75} = 12.$

2. (a) $\frac{4-z}{6} - z = \frac{z-5}{3} + 1$

$\Rightarrow$ $\frac{4-z-6z}{6} = \frac{z-5+3}{3}$

$\Rightarrow$ $3\,(4-7z) = 6\,(z-2)$

$\Rightarrow$ $12 - 21z = 6z - 12$

$\Rightarrow$ $-12z - 6z = -12 - 12 = -24$

$\Rightarrow$ $-27z = -24$

$\Rightarrow$ $z = \frac{-24}{-27} = \frac{8}{9}.$

(b) $\frac{a}{2} - \frac{29}{3} = \frac{3a-11}{4}$

$\Rightarrow$ $\frac{3a-4a}{6} = \frac{3a-11}{4}$

$\Rightarrow$ $4\,(-a) = 6\,(3a-11)$

$\Rightarrow$ $-4a = 18\,a \;\; -66$

$\Rightarrow$ $-4a - 18\,a = \;-66$

$\Rightarrow$ $-22\,a = \;-66$

$\Rightarrow$ $a = \frac{-66}{-22} = 3.$

(c) $x - \frac{x-1}{2} = 1 - \frac{x-2}{3}$

$\Rightarrow$ $\frac{2x-x+1}{2} = \frac{3x-x+2}{3}$

$\Rightarrow$ $3\,(2x - x + 1) = 2\,(-x + 5)$

$\Rightarrow$ $3\,(x+1) = -2x + 10$

$\Rightarrow$ $3x + 2x = 10 - 3$

$\Rightarrow$ $5x = 7$

$\Rightarrow$ $x = \frac{7}{5}.$

(d) $\frac{9m-7}{3m+3} = \frac{3m-4}{m+6}$

$\Rightarrow$ $(m+6)\,(9m-7) + (3m-4)\,(3m+3)$

$\Rightarrow$ $\cancel{9m^2} - 7m + 54m - 42 = \cancel{9m^2} + 9m - 12m - 12$

$\Rightarrow$ $+47\,m - 42 = -3m - 12$

$\Rightarrow$ $+47\,m + 3m = -12 + 42$

$\Rightarrow$ $50m = 30$

$\Rightarrow$ $m = \frac{30}{50} = \frac{3}{5}$.

(e) $\frac{x+6}{4} - \frac{5x-4}{8} = \frac{x-3}{2}$

$\Rightarrow$ $\frac{2x + 12 - 5x + 4}{8} = \frac{x-3}{2}$

$\Rightarrow$ $2(-3x + 16) = 8(x - 3)$

$\Rightarrow$ $-6x + 32 = 8x - 24$

$\Rightarrow$ $-6x - 8x = -24 - 32$

$\Rightarrow$ $-14x = -56$

$\Rightarrow$ $x = \frac{-56}{-14} = 4$.

(f) $\frac{-2(p-6)}{7} + p = \frac{p-3}{2}$

$\Rightarrow$ $\frac{-2p + 12 + 7p}{7} = \frac{p-3}{2}$

$\Rightarrow$ $2(5p + 12) = 7(p - 3)$

$\Rightarrow$ $10p + 24 = 7p - 21$

$\Rightarrow$ $10p - 7p = -21 - 24$

$\Rightarrow$ $3p = -45$

$\Rightarrow$ $p = \frac{-45}{3} = -15$.

3. Let the smaller number be x. Then the greater number = 2x + 10.
According to the question,
$x + 2x + 10 = 85$
$\Rightarrow$ $3x + 10 = 85$
$\Rightarrow$ $3x - 80 - 10 = 75$
$\Rightarrow$ $x = \frac{75}{3} = 25$.
$\therefore$ Greater number = $2x + 10 = 2 \times 25 + 10 = 60$.
Thus, the greater number is 60.

4. Let the side of the square be x cm.
Then the length of the rectangle = x cm and its breadth = $\frac{x}{2}$ cm.
Perimeter of the rectangle = 12 cm [Given]
$\Rightarrow$ 2 (length + breadth) = 12 cm
$\Rightarrow$ length + breadth = 6 cm
$\Rightarrow$ $x + \frac{x}{2} = 6m$
$\Rightarrow$ $2x + x = 12$ cm
$\Rightarrow$ $3x = 12$ cm
$\Rightarrow$ $x = 12$ cm $\div 3 = 4$ cm

$\therefore$ Side of the square = 4 cm

$\therefore$ Area of the square = 4 cm × 4 cm = 16 cm^2.

Thus, the area of the square is 16 cm^2.

5. Let the ages of Sonu and Raju be 2x and 3x respectively.

After five years : Age of Sonu = (2x + 5) years and Raju's age = (3x + 5) years

According to the question,

$$\frac{2x + 5}{3x + 5} = \frac{3}{4}$$

$\Rightarrow$ 8x + 20 = 9x + 15 [By cross multiplication]

$\Rightarrow$ 8x – 9x = 15 – 20

$\Rightarrow$ –x = –5

$\Rightarrow$ x = 5

$\therefore$ 2 x = 2 × 5 = 10 and 3x = 3 × 5 = 15.

Thus, ages of Sonu and Raju are respectively 10 years and 15 years.

6. Let the length of the rectangle be x cm.

Then its breadth = (x – 9) cm.

Increased length = (x + 3) cm

Increased breadth = (x – 9 + 3) = (x – 6) cm.

According to the question,

Area of new rectangle – area of original rectangle = 84

$\Rightarrow$ [(x + 3) (x – 6)] – [x × (x – 9)] = 84

$\Rightarrow$ $\cancel{x^2} - 3x - 18 - \cancel{x^2} + 9x = 84$

$\Rightarrow$ 6x = 84 + 18 = 102

$\Rightarrow$ x = 102 ÷ 6 = 17 cm

Hence, the length and breadth of the rectangle are respectively 17 cm and 8 cm.

7. Let the ones place digit of the number be x. Then its tens place digit = 9 – x

The number formed = 10 (9 – x) + x = 90 – 9x

On reversing the digits, new number = 10 x + (9 – x) = 10 x + 9 – x = 9x + 9

According to the question,

10 (9 – x) + x = (9x + 9) + 9

$\Rightarrow$ 90 – 10 x + x = 9x + 9 + 9

$\Rightarrow$ –18 x = –90 + 18

$\Rightarrow$ 18 x = 72

$\Rightarrow$ x = 72 ÷ 18 = 4

$\therefore$ 9 – x = 9 – 4 = 5

Thus, the required number is 54.

8. Let the three consecutive numbers be x, x + 1 and x + 2.

In decreasing order, numbers are x + 2, x + 1 and x.

According to the question,

7(x + 2) + 5 (x + 1) + 3 × x = 124

$\Rightarrow$ 7x + 14 + 5x + 5 + 3x = 124

$\Rightarrow$ 15 x + 19 = 124

$\Rightarrow$ 15 x = 124 – 19 = 105

$\Rightarrow$ x = 105 ÷ 15 = 7

$\therefore$ x + 1 = 7 + 1 = 8 and x + 2 = 7 + 2 = 9

Hence, the required consecutive numbers are 7, 8 and 9.

9. Let the present age of Monu be x years. Then the present age of Ruchir = 2 x years

Monu's age 11 years ago = (x – 11) years.

In six years, Ruchir's age will be (6 + 2x) years.

According to the question,

2x + 6 = 6 (x – 11)

$\Rightarrow$ 2x + 6 = 6x – 66

$\Rightarrow$ 2x – 6x = –66 – 6

$\Rightarrow$ –4 x = –72

$\Rightarrow$ x = 72 ÷ 4 = 18

$\therefore$ 2x = 18 × 2 = 36

Hence, the present age of Monu is 18 years and that of Ruchir is 36 years.

10. Distance travelled by car I in 1 h = 50 km

Distance travelled by car II in 1 hr = 60 km

Distance travelled by both the ears in 1 h = 50 km + 60 km = 110 km

Time taken by cars to travel 110 km = 1 h

$\therefore$ Time taken by cars to travel 1 km = $\frac{1}{110}$ h

$\therefore$ Time taken by cars to travel 550 km = $\frac{550}{110}$ = 5 h.

Hence, the cars will be 550 km a part in 5 hours.

11. Let the side of the square park be x m. Then its area = x^2 sq m.

New Width = (x + 2) m

New Length = (x + 5) m

Area of the new figure so formed = Length × breadth = (x + 5) (x + 2) = $x^2 + 7x + 10$

According to the question,

New area – Original area = 59 m

$\Rightarrow$ $\cancel{x^2} + 7x + 10 - \cancel{x^2} = 59$ m

$\Rightarrow$ 7x = 59 – 10 = 49 m

$\Rightarrow$ x = 49 ÷ 7 = 7 m

Hence, the length of the side of square park in 7 m.

Multiple Choice Questions

1. Let the present age of Rahul be x years

Then after ten years his age = (x + 10) years

According to the question,

x + 10 = 2x

$\Rightarrow$ x – 2x = –10

$\Rightarrow$ $-x = -10$

$\Rightarrow$ $x = 10$, which is Rahul's present age.

Rahul's age 5 years ago = 10 – 5 = 5 years

Hence, the correct option is (b).

2. Let the present age of Suresh be x years.

   Then after eight years, his age = (x + 8) years

   According to the question,

   $x + 8 = 5x$

   $\Rightarrow$ $x - 5x = -8$

   $\Rightarrow$ $-4x = -8$

   $\Rightarrow$ $x = 8 \div 4 = 2$

   Hence, the correct option is (d).

3. Similar work to be done. Correct option is (c).

4. Let the angles of the triangle be 2x, 3x and 4x. Then by angle sum property of the triangle,

   $2x + 3x + 4x = 180^o$

   $\Rightarrow$ $9x = 180^o$

   $\Rightarrow$ $x = 180^o \div 9 = 20^o$

   $\Rightarrow$ $2x = 2 \times 20^o = 40^o$

   Hence, the correct option is (c).

5. Let the number thought be x. Then according to the question.

   $\left(x - \frac{3}{4}\right) \times 4 = 3x$

   $\Rightarrow$ $\left(\frac{4x - 3}{4}\right) \times 4 = 3x$

   $\Rightarrow$ $16x - 12 = 4 \times 3x$

   $\Rightarrow$ $16x - 12x = 12$

   $\Rightarrow$ $4x = 12$

   $\Rightarrow$ $x = 12 \div 4 = 3$

   Hence, the correct option is (d).

6. $3y - 5 = y - 3$

   $\Rightarrow$ $3y - y = -3 + 5$ $\Rightarrow$ $2y = 2$ $\Rightarrow$ $y = 1$.

   Hence, the correct option is (a).

7. $a = 2x - 5$

   $\Rightarrow$ $9 + 5 = 2x$ $\Rightarrow$ $14 = 2x$ $\Rightarrow$ $x = 14 \div 2 = 7$

   Hence, the correct option is (b)

8. $\frac{m}{3} = \frac{-5}{2} - \frac{3}{2} = \frac{-5 - 3}{2} = \frac{-8}{2} = -4$

   $\Rightarrow$ $\frac{m}{3} = -4$ $\Rightarrow$ $m = -4 \times 3 = -12$.

   Hence, the correct option is (d).

9. Let the required consecutive multiples of 4 be x and x + 4.

Then according to the question,

$x + x + 4 = 68$

$\Rightarrow 2x = 68 - 4 = 64 \quad \Rightarrow x = 64 \div 2 = 32$

Hence, the correct option is (c).

10. Area of the rectangle = Perimeter of the rectangle

$\Rightarrow$ length × breadth = 2 (length + breadth)

$$\Rightarrow 7\frac{1}{3}\text{cm} \times \text{breadth} = 2\left(7\frac{1}{3}\text{cm} + \text{breadth}\right)$$

$$\Rightarrow \frac{22}{3}\text{cm} \times \text{breadth} = 2\left(\frac{22}{3}\text{cm} + \text{breadth}\right)$$

$$\Rightarrow \text{breadth}\left(\frac{22}{3} - \frac{6}{3}\right) = \frac{44}{3}$$

$\Rightarrow 16 \times \text{breadth} = 44$

$$\Rightarrow \text{breadth} = \frac{\cancel{44}^{11}}{\cancel{16}_4} = \frac{11}{4} = 2\frac{3}{4}.$$

Hence, the correct option is (b).

Mental Maths

A. See the **Answers** given in the book.

B. See the **Answers** given in the book.

Higher Order Thinking Skills (HOTS)

1. $8\ |x| = 40$

$\Rightarrow 8 \times x = 40$

$\Rightarrow x = 40 \div 8 = 5.$

Thus, the value of x is 5.

2. Let the number of bananas Ravi got be x. Then the number of bananas Rakesh got = 2 x.

According to the question.

$x + 2x = 12$ [$\because$ 1 dozen = 12]

$\Rightarrow 3x = 12$

$\Rightarrow x = 12 \div 3 = 4.$

Hence, Ravi got 4 bananas and Rakesh got 8 bananas.

3. Let the number of students in the group be x.

Then the number of student watching TV = $\frac{x}{2}$

Number of students who are playing = $\frac{x}{2} \times \frac{3}{4} = \frac{3x}{8}.$

According to the question,

$$x - \left(\frac{x}{2} + \frac{3x}{8}\right) = 18 \qquad \Rightarrow x - \left(\frac{4x - 3x}{8}\right) = 18$$

$\Rightarrow 8x - 7x = 18 \times 8 = 144 \qquad \Rightarrow x = 144.$

Here, there are 144 students in the group.

Higher Order Thinking Skills (HOTS)

1. Given : $3x = 27$ $\quad \therefore\ x = 27 \div 3 = 9$

   Now, $x = 3y$

   $\Rightarrow\ 9 = 3y \quad \Rightarrow\ 9 \div 3 = y \quad \Rightarrow\ y = 3.$

   Thus, the value of y is 3.

2. Given : $\frac{4x}{3}$ [ ] 4 = 16 and value of x = 3.

   $\therefore\ \frac{4x}{3}$ [ ] 4 = 16

   $\Rightarrow\ \frac{4 \times 3}{3}$ [ ] 4 = 16

   $\Rightarrow$ 4 [ ] 4 = 16 $\quad \Rightarrow\ 4 \times 4 = 16.$ [Substituting the value of x]

   Thus, the required sign is multiplication sign (x).

3. $\frac{5x + 3}{3} = 11$

   $\Rightarrow\ 5x + 3 = 11 \times 3 = 33$

   $\Rightarrow\ 5x = 33 - 3 = 30 \quad \Rightarrow\ x = 30 \div 5 = 6$

   Now, $5x + 3 = 5 \times 6 + 3 = 33.$

4. $\frac{2y - 2}{7y - 1} = \frac{2}{3}$

   $\Rightarrow\ 3(3y - 2) = 2\,(7y - 1)$ [By cross multiplication]

   $\Rightarrow\ 9y - 6 = 14y - 2$

   $\Rightarrow\ 9y - 14y = -2 + 6$

   $\Rightarrow\ -5y = 4 \quad \Rightarrow\ y = \frac{4}{5}$

5. $3x^2 \times 2 = 241$

   LHS = $3x^2 \times 2 = 3 \times (-9)^2 \times 2 = 3 \times 81 \times 2 = 243 \times 2 = 243 - 2 = 241.$

   Thus, the mathematical sign that replace × will be minus sign (–).

6. $\frac{2}{5x - 2} = \frac{-1}{11} \quad \Rightarrow\ 2 \times 11 = -1 \times (5x - 2)$

   $\Rightarrow\ 22 = -5x + 2 \ \Rightarrow\ 5x = -22 + 2 = -20 \quad \Rightarrow\ x = -20 \div 5 = -4.$

   $\therefore\ 2x + 3 = 2 \times (-4) + 3 = -8 + 3 = -5.$

   Thus, the value of 2x + 3 is –5.

9 Comparing Quantities

Exercise 8.1

1. (a) 36% of 480 g = $\frac{36}{100} \times 480$ g. = 1728 g.

   (b) 45% of 1200 km = $\frac{45}{\cancel{100}} \times 12\cancel{00}$ km = 45 × 12 km = 540 km.

(c) 40% of 5 km = $\frac{40}{100} \times 5$ km = $\frac{200}{100}$ km = 2 km.

(d) 10% of Rs. 2 = $\frac{10}{100} \times 200$ p = 20 p or Rs. 0.2.

2. (a) $\frac{1}{4}\%$ of x = 36

$\frac{1}{4 \times 100} \times x = 36$

x = 36 × 400 = 14400.

Thus, the value of x is 14400.

(b) 2.5% of x = 20

$\frac{2.5}{100} \times x = 20$

2.5 x = 20 × 100

$x = \frac{20 \times 100}{2.5} = 800.$

Thus, the value of x is 800.

(c) 12.5% of x = 5

$\frac{12.5}{100} \times x = 5$

$x = \frac{5 \times 100}{12.5} = 40.$

Hence, the value of x is 40.

(d) 60% of x = 324

$\frac{60}{100} \times x = 324$

$x = \frac{324 \times 100}{60} = 540.$

Hence, the value of x is 540.

3. (a) Let x% of 5 kg be 125 g

Then $\frac{x}{100} \times 5000\,g = 125\,g$

$x = \frac{125 \times 100}{5000} = 2.5\%$

Hence, 125 g is 2.5% of 5 kg.

(b) Let x% of 1 km be 40 m

Then $\frac{x}{100} \times 1000\,m = 40\,m$

$x = \frac{\cancel{40} \times \cancel{100}}{\cancel{1000}} = 4\%$

Hence, 40 m are 4% of 1 km.

(c) Let x% of Rs. 2 be 25 p.

Then $\frac{x}{100} \times 200\,p = 25$

$x = \frac{25 \times 100}{200} = 12.5\%$

Hence, 25 p are 12.5% of ₹ 2.

(d) Let x% of 1 day be 6 h.

Then $\frac{x}{100} \times 24 = 6$

$x = \frac{\cancel{6}^{1} \times 100}{\cancel{24}_{4}} = \frac{100}{4} = 25\%$

Hence, 6 h are 25% of 1 km.

4. Let the monthly income of Suman be ₹ x.

Then 25% of x = ₹ 2500.

$\frac{25}{100} \times x$ = ₹ 2500.

x = ₹ $\frac{\cancel{2500}^{100} \times 100}{\cancel{25}_{1}}$ = ₹ 10000.

Hence, the monthly income of Suman is ₹ 10000.

5. Let the total number of students in the class be x.

Then 30% of x = 60

$\Rightarrow \frac{30}{100} \times x = 60 \quad \Rightarrow x = \frac{\cancel{60}^{2} \times 100}{\cancel{30}_{1}} = 200.$

Hence, the total number of students in class VIII is 200.

6. Let the volume of the milkshake be x.

Then the quantity of milk in the milkshake = 40% of x = $\frac{40}{100} \times x = \frac{2x}{5}$

Quantity of water in the milkshake = $x - \frac{2x}{5} = \frac{5x - 2x}{5} = \frac{3x}{5}$

According to the question,

$\frac{3x}{5} - \frac{2x}{5} = 400$ mL

$\therefore$ x = 400 × 5 = 2000 mL = $\frac{2000}{1000}$L = 2L

Hence, the volume of the milkshake is 2L.

7. Total number of fruits in the box = 800

Number of oranges = 20% of 800 = $\frac{20}{\cancel{100}_1} \times \cancel{800}^8 = 160$.

Number of apples = 15% of 800 = $\frac{15}{\cancel{100}_1} \times \cancel{800}^8 = 120$.

Number of guavas = 800 – (160 + 120) = 800 – 280 = 520.

Hence, there are 160 oranges, 120 apples and 520 guavas in the box.

8. Let the marks that Sunanda could have got be x.

Then 70% of x = 370 + 15 = 385

$\frac{70}{100} \times x = 385$

$\therefore$ x = $\frac{385 \times 10\cancel{0}}{7\cancel{0}}$ = 55 × 10 = 550 marks.

Hence, Sunanda could have got 550 marks.

9. (a) Number of runs scored by sixes = 4 × 6 = 24 runs

Percentage = $\frac{24}{75} \times 100 = 32\%$

(b) Runs scored by fours = 10 × 4 = 40 runs

Percentage = $\frac{40}{\cancel{75}_3} \times \cancel{100}^4 = \frac{160}{3} = 53\frac{1}{3}\%$

(c) Runs scored by twos = 4 × 2 = 8

Percentage = $\frac{8}{\cancel{75}_3} \times \cancel{100}^4 = \frac{32}{3} = 10\frac{2}{3}\%$.

(d) Runs scored by singles = 3

Percentage = $\frac{\cancel{3}}{\cancel{75}_{25}} \times 100 = \frac{\cancel{100}^4}{\cancel{25}_1} = 4\%$.

10. Let Anita's income be ₹ 100.

Then Sunita's income = ₹ 100 – 10% of ₹ 100 = ₹ 90.

Difference between the both incomes = ₹ 100 – ₹ 90 = ₹ 10.

Percentage = $\frac{\cancel{10} \times 100}{\cancel{90}_9} = \frac{100}{9} = 11\frac{1}{9}\%$.

Hence, Anita's income is $11\frac{1}{9}\%$ more than Sunita.

11. Let the price of sugar be ₹ 100.
The increased price of sugar = ₹ 100 + 25% of ₹ 100 = ₹ 100 + ₹ 25 = ₹ 125.
Difference = ₹ 125 – ₹ 100 = ₹ 25.
Percentage consumption to be reduced = $\frac{\cancel{25}^{1}}{\cancel{125}_{5}} \times 100\% = \frac{100}{5}\% = 20\%$.

Thus, the consumption must be reduced by 20%.

12. Let the total number of votes polled be x.
Then 52% of x = $\frac{\cancel{52}^{13}}{\cancel{100}_{25}} \times x = \frac{13x}{25}$.
48% of x = $\frac{\cancel{48}^{12}}{\cancel{100}_{25}} \times x = \frac{12x}{25}$.
Difference = $\frac{13x}{25} - \frac{12x}{25} = \frac{x}{25}$.
$\therefore \frac{x}{25} = 9375$
x = 9375 × 25 = 234375.
Hence, the total number of votes polled is 234375.

13. Let the machines value last year be x.
Then 10% of x = $\frac{10}{100} \times x = \frac{x}{10}$
Present value of the machine = ₹ 18000.
$x - \frac{x}{10} =$ ₹ 18000.
$\Rightarrow \frac{10x - x}{10} =$ ₹ 18000.
$\Rightarrow \frac{9x}{10} =$ ₹ 18000. $\Rightarrow x = \frac{18000 \times 10}{9} =$ ₹ 20000.
Hence, the value of the machine last year was ₹ 20000.

14. Let the money distributed between A and B be ₹ x.
Then share of A = 70% of x = $\frac{7\cancel{0}}{10\cancel{0}} \times x = \frac{7x}{10}$
30% of x = $\frac{3\cancel{0}}{10\cancel{0}} \times x = \frac{3x}{10}$
Difference = $\frac{7x}{10} - \frac{3x}{10} = \frac{4x}{10}$.
$\therefore \frac{4x}{10} =$ ₹ 2500
$\Rightarrow$ 4x = ₹ 2500 × 10
x = ₹ $\frac{25000}{4}$ = ₹ 6250.
Hence, ₹ 6250 has distributed between A and B.

Exercise 9.2

1. CP of sofa = ₹ 4800

   Overhead expenses = ₹ 350 + ₹ 250 = ₹ 600

   Actual CP of sofa = ₹ 4800 + ₹ 600 = ₹ 5400

   SP of sofa = ₹ 5200

   As SP is less than CP, so there is a loss.

   Loss = CP – SP = ₹ 5400 – ₹ 5200 = ₹ 200.

   $$\text{Loss\%} = \frac{\text{Loss}}{\text{CP}} \times 100 = \frac{200}{5400} \times 100 = \frac{200\cancel{00}}{54\cancel{00}} = \frac{200}{54} = \frac{100}{27} = 3\frac{19}{27}\%$$

   Hence, the loss percent of Surjeet is $3\frac{19}{27}\%$.

2. SP of the plot of land = ₹ 3,60,000

   Profit earned = 20%

   $$\therefore \text{CP of the plot} = \frac{\text{SP} \times 100}{100 + \text{Profit\%}} = ₹\ \frac{3,60,000 \times 100}{120} = ₹\ 3,00,000.$$

   Hence, the CP of the plot of land is = ₹ 3,00,000.

3. Loss = CP of 40m pipe = SP of 40m pipe

   SP of 4 m pipe = CP of 40 m pipe – SP of 40 m pipe.

   SP of 4 m pipe + SP of 40 m pipe = CP of 40 m pipe

   SP of 44 m pipe = CP of 40 m pipe

   Let CP of 1 m pipe be x.

   Then CP of 40 m pipe = 40x.

   ∴ SP of 44 m pipe = CP of 40 m pipe = 40 x.

   ∴ CP = 44 m and SP = 40 m

   Loss = 44 x – 40 x = 4x

   $$\text{Loss\%} = \frac{\text{loss}}{\text{CP}} \times 100\% = \frac{4x}{44x} \times 100\% = \frac{100}{11} = 9\frac{1}{11}\%$$

   Hence, the loss percentage is $9\frac{1}{11}\%$.

4. Let the CP of the shirt be ₹ 100.

   Then its loss = 15% of ₹ 100 = $\frac{15}{\cancel{100}} \times \cancel{100}$ = ₹ 15.

   ∴ SP of the shirt = ₹ 100 – ₹ 15 = ₹ 85.

   When SP is ₹ 85, then CP of the shirt = ₹ 100

   When SP is ₹ 1, then CP of the shirt = ₹ $\frac{100}{85}$

   When SP is ₹ 1700, then CP of the shirt = ₹ $\frac{1700 \times 100}{85}$ = ₹ 20 × 100 = ₹ 2000.

   Now, the required profit = 20%. [Given]

   $$\therefore \text{SP} = \frac{100 + \text{Profit}}{100} \times \text{CP} = ₹\ \frac{100 + 20}{1\cancel{00}} \times 20\cancel{00} = ₹\ 120 \times 20 = ₹\ 2400.$$

   Hence, the selling price of the shirt should be ₹ 2400.

5. SP of 1 banana at ₹ 72 per dozen = ₹ $\frac{72}{12}$ = ₹ 6.

Let CP of 1 banana be ₹ x

Then CP of 1 banana = $\frac{SP \times 100}{100 - loss\%}$ = ₹ $\frac{6 \times 100}{100 - 10} = \frac{600}{90} = \frac{60}{9}$ = ₹ $\frac{20}{3}$

Now, SP of 1 banana at ₹ 700 per hundred = ₹ $\frac{700}{100}$ = ₹ 7.

As ₹ 7 > ₹ $\frac{20}{3}$, so there is a profit (gain).

Gain = ₹ 7 – ₹ $\frac{20}{3}$ = ₹ $\frac{21-20}{3}$ = ₹ $\frac{1}{3}$.

$$\text{Gain\%} = \frac{\text{Gain}}{\text{CP}} \times 100 = \frac{\frac{1}{3} \times 100}{\frac{20}{3}} = \frac{100 \times \cancel{3}}{20 \times \cancel{3}}\% = 5\%.$$

Hence, Suhel gets a gain of 5% in this transaction.

6. CP of double bed = ₹ 8500.

Overhead expenses = ₹ (500 + 400) = ₹ 900.

Actual CP of double bed = ₹ (8500 + 900) = ₹ 9400.

SP of the double bed = ₹ 9600.

As SP is greater than CP, so there is a gain.

∴ Gain = ₹ 9600 – ₹ 9400 = ₹ 200.

$$\text{Gain\%} = \frac{\text{Gain}}{\text{CP}} \times 100\% = \left(\frac{200}{94\cancel{00}} \times 1\cancel{00}\right)\% = \frac{200}{94}\% = \frac{100}{47}\% = 2\frac{6}{47}\%$$

Hence, Rekha got a profit of $2\frac{6}{47}$%.

7. Let CP of 1 apple be ₹ 1.

Then CP of 10 apples = ₹ 10.

CP of 9 apples = ₹ 9.

According to the question.

SP of a apples = CP of 10 apples = ₹ 10.

As SP is greater than CP, then there is a gain

Gain = SP – CP = ₹ 10 – ₹ 9 = ₹ 1.

$$\text{Gain\%} = \left(\frac{\text{Gain}}{\text{CP}} \times 100\right)\% = \left(\frac{1}{9} \times 100\right)\% = \frac{100}{9} = 11\frac{1}{9}\%.$$

Hence, the required gain percentage is $11\frac{1}{9}$%.

8. Let CP of a cooler = ₹ 1200.

Overhead expenses = ₹ 100.

Actual CP of cooler = ₹ 1200 + ₹ 100 = ₹ 1300.

Profit = 8% [Given]

∴ SP of the cooler = $\frac{100 + \text{Profit\%}}{100} \times \text{CP}$ = ₹ $\frac{100 + 8}{1\cancel{00}}$ × ₹ 13$\cancel{00}$ = ₹ (108 × 13) = ₹ 1404.

Hence, the selling price of the cooler is ₹ 1404.

9. For one computer

Given : SP of computer = ₹ 19800 and gain = 10%

$\therefore$ CP of the computer = $\frac{SP \times 100}{100 + Gain}$ = ₹ $\frac{19800 \times 100}{100 + 10}$ = ₹ $\frac{19800 \times 100}{110}$ = ₹ 18000.

For another computer

Given : SP of computer = ₹ 19800 and loss = 10%

$\therefore$ CP of the computer = $\frac{SP \times 100}{100 - Loss}$ = ₹ $\frac{19800 \times 100}{100 - 10}$ = ₹ $\frac{19800 \times 100}{90}$ = ₹ 22000.

CP of both the computers = ₹ 18000 + ₹ 22000 = ₹ 40000.

SP of both the computers = ₹ 19800 + ₹ 19800 = ₹ 39600.

As CP is greater than SP, so there is a loss.

Loss = CP – SP = ₹ 40000 + ₹ 39600 = ₹ 400.

Loss% = $\frac{Loss}{CP} \times 100 = \frac{400 \times 100}{40000}\% = 1\%$

Hence, the loss percentage in the whole transaction is 1%.

10. Let the CP of the article be ₹ x.

Then profit at 6% = 6% of x = $\frac{6}{100} \times x = \frac{6x}{100}$

SP of the article = $x + \frac{6x}{100} = \frac{106x}{100}$

Profit when article is sold at 4% = 4% of x = $\frac{4}{100} \times x = \frac{4x}{100}$

SP of the article = $x + \frac{4x}{100} = \frac{104x}{100}$

According to the question,

Difference between the two SPs = ₹ 3

$\Rightarrow \frac{106x}{100} - \frac{104x}{100}$ = ₹ 3

$\Rightarrow \frac{2x}{100}$ = ₹ 3

$\Rightarrow$ x = ₹ $\frac{300}{2}$ = ₹ 150.

$\therefore$ SP when the article is sold at 6% profit = $x + \frac{6x}{100}$ = ₹ 150 + ₹ $\frac{6 \times 150}{100}$ = ₹ 150 + ₹ 9 = ₹ 159.

$\therefore$ SP when the article is sold at 4% profit = $x + \frac{4x}{100}$ = ₹ 150 + ₹ $\frac{4 \times 150}{100}$ = ₹ 150 + ₹ 6 = ₹ 156.

11. Let the CP of the memory card be ₹ x.

Gain = 5% [Given]

$\therefore$ SP of the memory card = ₹ $x + \frac{5x}{100} = \frac{21x}{20}$.

Loss = 2% [Given]

$\therefore$ SP of the memory card = x – 2% of x = $x - \frac{2x}{100} = x - \frac{x}{50} = \frac{49x}{50}$.

According to the question,

$$\frac{21x}{20} - \frac{49x}{50} = ₹\ 63.$$

$$\Rightarrow \frac{105x}{100} - \frac{98x}{100} = ₹\ 63.$$

$\Rightarrow 7x = ₹\ 63 \times 100$

$\Rightarrow x = ₹\ 6300 \div 7 = ₹\ 900.$

Hence, the SP of the memory card is ₹ 900.

12. Let the cost of production of the juicer be ₹ x.

Then SP of the juicer for the manufacturer = 120% of ₹ x

$$= ₹\ \frac{120 \times x}{100} = ₹\ \frac{6x}{5}$$

Whole seller gains = 10% [Given]

$$\therefore \text{ SP of the juicer for the wholeseller} = 110\% \text{ of } ₹\ \frac{6x}{5} = \frac{6x}{5} \times \frac{11\cancel{0}}{10\cancel{0}} = \frac{66x}{50}$$

Now, retailer's gain = 12.5%

$$\therefore \text{ SP of the juicer for retailer} = 112.5\% \text{ of } ₹\ \frac{66x}{50} = ₹\ \frac{66x}{50} \times \frac{112.5}{100}$$

$$\therefore \frac{66x}{50} \times \frac{112.5}{100} = ₹\ 2673. \qquad \Rightarrow x = ₹\ \frac{2673 \times 50 \times 100}{112.5 \times 66} = ₹\ 18,00.$$

Hence, the cost of production of the juicer is ₹ 1800.

Exercise 9.3

1. (a) Discount = MP – SP = ₹ 1500 – ₹ 1200 = ₹ 300.

$$\text{Discount\%} = \frac{\text{Discount}}{\text{MP}} \times 100\% = \frac{300}{1500} \times 100 = \frac{300}{150\cancel{0}} \times 10\cancel{0} = \frac{300}{15} = 20\%$$

(b) Discount = MP – SP = ₹ 480 – ₹ 440 = ₹ 40.

$$\text{Discount\%} = \frac{\text{Discount}}{\text{MP}} \times 100\% = \frac{40}{480} \times 100\% = \frac{400\cancel{0}}{48\cancel{0}} = \frac{400}{48} = \frac{50}{6} = \frac{25}{3}\% = 8\frac{1}{3}\%.$$

2. (a) Amount paid for medicine = ₹ 7200 + 6% of ₹ 720 = ₹ 720 + ₹ 43.20 = ₹ 763.20.

(b) Amount paid for trouser = ₹ 1190 + 4% of ₹ 1190 = ₹ 1190 + ₹ 47.60 = ₹ 1237.60.

(c) Amount paid for cosmetics = ₹ 425 + 8% of ₹ 425 = ₹ 425 + ₹ 34 = ₹ 459.00.

(d) Amount paid for watch = ₹ 2200 + 10% of ₹ 2200 = ₹ 2200 + ₹ 220 = ₹ 2420.

∴ Total amount paid by Poonam = ₹ 763.20 + 1237.60 + ₹ 459.00 + ₹ 2420 = ₹ 4879.40.

3. Let MP of the article be ₹ x.

$$\text{Then its SP} = x - 6\% \text{ of } x = x - \frac{6}{100} \times x = x - \frac{6x}{100} = \frac{94x}{100}$$

According to the question,

$$\frac{94x}{100} = ₹\ 611 \qquad \Rightarrow x = ₹\ \frac{611 \times 100}{94} = ₹\ 650$$

Hence, the marked price of the article is ₹ 650.

4. Given : Marked price = ₹ 950 and discount = 35%.

Discount = 35% of ₹ 950 = $\frac{35}{10\cancel{0}} \times$ ₹ $95\cancel{0}$ = ₹ $\frac{35 \times 95}{10}$ = ₹ 332.50

$\therefore$ Selling price = Marked price – Discount = ₹ 950 – ₹ 332.50 = ₹ 617.50.

Hence, the selling price is ₹ 617.50.

5. Given : CP of DVD = ₹ 2450 and VAT = 8%.

$\therefore$ Amount paid by Rajesh = CP + Discound = ₹ 2450 + 8% of ₹ 2450 = ₹ 2450 + ₹ 196 = ₹ 2646.

Hence, the amount paid by Rajesh is ₹ 2646.

6. Marked price (MP) of the dress = ₹ 4275 [Given]

Let the discount be ₹ x. Then SP of the dress = ₹ (4275 – x)

VAT applied = 12.5%

$\therefore$ SP + VAT = 4275 = $(4275 - x) + \frac{12.5}{100} \times (4275 - x) = 4275$

$\Rightarrow (4275 - x)\left[1 + \frac{1}{8}\right] = 4275$

$\Rightarrow (4275 - x) \times \frac{9}{8} = 4275$

$\Rightarrow 4275 - x = \frac{4275 \times 8}{9} = 3800$

$\Rightarrow 4275 - 3800 = x$

$\therefore x = 475$

Now discount% = $\frac{475}{4275} \times 100 = 11.11\%$

Hence, the selling price of the dress is ₹ 3800 and discount percent is 11.11%.

7. Given : MP = ₹ 1800, discount = 10% and profit = 20%.

As discount = 10%

$\therefore$ SP = 90% of MP = $\frac{90}{100}$ × ₹ 1800 = ₹ (90 × 18) = ₹ 1620.

Given profit = 20%.

$\therefore$ SP = 120% of CP

$\Rightarrow$ ₹ 1620 = $\frac{120}{100} \times$ CP

$\Rightarrow$ CP = ₹ $\frac{1620 \times 100}{120}$ = ₹ 1350.

Hence, the cost price of the item is ₹ 1350.

8. Let CP be ₹ 100.

Then profit% = $\frac{SP - CP}{CP} \times 100$

$\Rightarrow 5 = \left(\frac{SP - 100}{100}\right) \times 100$ [Given profit = 5%]

SP = ₹ 105

Discount = 16% [Given]

$\therefore SP = \frac{MP(100 - Discount)}{100}$

$\Rightarrow 105 = \dfrac{MP(100 - \text{Discount})}{100}$

$\Rightarrow MP = \dfrac{105 \times 100}{100 - 16} = \dfrac{10500}{84} = 125.$

$\Rightarrow MP - CP = 125 - 100 = 25.$

$\therefore$ Percentage of MP = $\left(\dfrac{25}{\cancel{100}} \times \cancel{100}\right)\% = 25\%.$

Hence, the required percentage of marked price is 25%.

9. Given : MP of the air cooler = ₹ 5500.

$\therefore$ SP of the cooler after first discount = ₹ 5500 – 10% of ₹ 5500 = ₹ 5500 – ₹ 550 = ₹ 4950.

$\therefore$ SP of the cooler after second discount =₹ 4950 – 5% of ₹ 4950 = ₹ 4950 – ₹ 247.50 = ₹ 4702.50.

Hence, the selling price of the air cooler is ₹ 4702.50.

10. Given : CP of the article = ₹ 500, profit = 60% of CP and discount = 15% on MP.

(a) MP of the article = ₹ 500 + 60% of ₹ 500 = ₹ 500 + ₹ 300 =₹ 800.

(b) Discount given to the customer = 15% of ₹ 800 = $\dfrac{15}{100}$ × ₹ 800 = ₹ 120.

(c) Actual discount % = $\dfrac{120 \times 100}{500}\% = 24\%$

$\therefore$ Actual profit made by the shopkeeper = 60% – 24% = 36%.

11. Let the marked price of jeans be ₹ x.

Then VAT = 10% of x = $\dfrac{10}{100} \times x$ = ₹ $\dfrac{x}{10}$.

Price including VAT = $x + \dfrac{x}{10} = \dfrac{11x}{10}$

$\therefore \dfrac{11x}{10}$ = ₹ 770 $\Rightarrow$ x = ₹ $\dfrac{770 \times 10}{11}$ = ₹ 700.

$\therefore$ MP of jeans = ₹ 700.

Let the marked price of the jacket be ₹ y.

Then VAT = 12% of y = $\dfrac{\cancel{12}^{3}}{\cancel{100}_{25}} \times y = \dfrac{3y}{25}$

Price including VAT = $x + \dfrac{3y}{25} = \dfrac{28y}{25}$

$\therefore \dfrac{28y}{25}$ = ₹ 4480. $\Rightarrow$ y = ₹ $\dfrac{4480 \times 25}{28}$ = ₹ 4000.

$\therefore$ Total price of both the items = ₹ 700 + ₹ 4000 = ₹ 4700.

12. Let CP of the article be ₹ 100 and its MP be ₹ x.

Then gain percent after discount = 20% + 8% = 28%

Gain = 8% of ₹ 100 = ₹ 8.

SP of the article = CP + gain

₹ (100 + 20) = ₹ 100

Discount = 20% of MP = $\dfrac{20}{100} \times x = \dfrac{x}{5}$

According to the question,
MP – Discount = SP

$$\Rightarrow x - \frac{x}{5} = 108$$

$$\Rightarrow \frac{4x}{5} = 108$$

$$\Rightarrow x = \frac{108 \times 5}{4} = ₹\ 135.$$

$\therefore$ Amount marked above CP = MP – CP = ₹ 135 – ₹ 100 = ₹ 35.

$\therefore$ Percentage of amount marked $= \left(\frac{35}{100} \times 100\right)\% = 35\%$

Hence, the shopkeeper should mark the articles 35% above the cost price.

Revision Exercise

1. SP of both the watches = ₹ 1920 × 2 = ₹ 3840.
Let CP of each watch be ₹ x.

Then gain = 20% of x $= \frac{20}{100} \times ₹\ x = ₹\ \frac{x}{5}$

SP $= x + \frac{x}{5} = ₹\ \frac{6x}{5}$

$\therefore \frac{6x}{5} = ₹\ 1920$

$x = ₹\ \frac{1920 \times 5}{6} = ₹\ 320 \times 5 = ₹\ 1600.$

Now loss = 20% of x $= \frac{20}{100} \times ₹\ x = ₹\ \frac{x}{5}$

SP $= x - \frac{x}{5} = \frac{4x}{5}$

$\therefore \frac{4x}{5} = ₹\ 1920$

$x = ₹\ \frac{1920 \times 5}{4} = ₹\ 480 \times 5 = ₹\ 2400.$

Total CP of two watches = ₹ 1600 + ₹ 2400 = ₹ 4000.
As CP is greater than SP, so there is a loss.

Loss% $= \frac{\text{Loss}}{\text{CP}} \times 100 = \frac{4000 - 3840}{4000} \times 100 = \frac{16\cancel{0} \times 1\cancel{00}}{4\cancel{000}} = 4\%$

Hence, there is a loss of 4% in the whole transaction.

2. Let the list price of the LED set be ₹ x.

Then VAD = 12% of ₹ x = ₹ $\frac{12}{100} \times x = ₹\ \frac{3x}{25}$

List price including VAD = ₹ x + ₹ $\frac{3x}{25} = ₹\ \frac{28x}{25}$

According to the question,

₹ $\frac{28x}{25} = ₹\ 31920.$

$\therefore$ x = ₹ $\dfrac{31920 \times 25}{28}$ = ₹ (1140 × 25) = ₹ 28,500.

Hence, the list price of the LED set is ₹ 28500.

3. Given : Price of the car including VAT = ₹ 515200 and its basic price = ₹ 460000.

$\therefore$ VAT = ₹ 515,200 – ₹ 4,60,000 = ₹ 55,200.

$$\text{VAT\%} = \frac{\text{VAD}}{\text{Basic price}} \times 100 = \frac{55,200}{4,60,000} \times 100 = 12\%$$

Hence, the VAT percent is 12%.

4. Let CP be ₹ x.

Then discount = 10% of x $= \dfrac{10}{100} \times x = \dfrac{x}{10}$

Remaining amount $= x - \dfrac{x}{10} = \dfrac{9x}{10}$.

Again next discount = 5% of $\dfrac{9x}{10} = \dfrac{5}{100} \times \dfrac{9x}{10} = \dfrac{9x}{200}$

Total discount $= \dfrac{9x}{10} + \dfrac{9x}{200} = \dfrac{29x}{200}$

Single discount = 15% of x $= \dfrac{15}{100} \times x = \dfrac{3x}{20} = \dfrac{30x}{200}$

As $\dfrac{30x}{200} > \dfrac{29x}{200}$

Hence, the single discount of 15% is better.

5. **By Ist condition :**

CP of 11 marbles = ₹ 10

$\therefore$ CP of 1 marble = ₹ 10 ÷ 11 = ₹ $\dfrac{10}{11}$.

By IInd condition :

CP of 9 marbles = ₹ 10

$\therefore$ CP of 1 marble = ₹ $\dfrac{10}{9}$

$\therefore$ Total CP of both the marbles = ₹ $\dfrac{10}{11}$ + ₹ $\dfrac{10}{9}$ = ₹ $\dfrac{200}{99}$

$\therefore$ CP of 1 marble = ₹ $\dfrac{200}{99}$ √ 2 = ₹ $\dfrac{100}{99}$

SP of 1 marble = ₹ 1 [Given]

As CP in greater than SP, so there is a loss.

Loss = ₹ $\dfrac{100}{99}$ – ₹ 1 = ₹ $\dfrac{1}{99}$

$$\text{Loss\%} = \frac{\text{Loss}}{\text{CP}} \times 100 = \frac{1}{99} \div \frac{100}{99} \times 100 = \frac{1}{\cancel{99}} \times \frac{\cancel{99}}{\cancel{100}} \times \cancel{100} = 1\%$$

Hence, the loss percent in the whole transaction is 1%.

6. Given : CP of 20 cricket bats = ₹ 12,000 and profit = SP of 4 bats.

$\therefore$ Profit = SP of 20 bats – CP of 20 bats

SP of 4 bats = SP of 20 bats – CP of 20 bats

SP of 16 bats = CP of 20 bats

SP of 16 bats = ₹ 12,000

SP of 1 bat = ₹ $\frac{12000}{16}$ = ₹ 750.

Hence, SP of 1 bat is ₹ 750.

7. CP of 1 orange of first type = ₹ 30 ÷ 3 = ₹ 10.

CP of 1 orange of second type = ₹ 48 ÷ 4 = ₹ 12.

CP of there two oranges = ₹ 10 + ₹ 12 = ₹ 22.

SP of 5 oranges = ₹ 56.

$\therefore$ SP of 1 orange = ₹ $\frac{56}{5}$

$\therefore$ SP of 2 oranges = ₹ $\frac{56}{5} \times 2$ = ₹ $\frac{112}{5}$ = ₹ 22.4

As SP of 2 oranges > CP of 2 oranges, so there is a gain.

Gain = SP – CP = ₹ 22.4 – ₹ 22 = ₹ 0.40.

Gain% = $\frac{\text{Gain}}{\text{CP}} \times 100 = \frac{0.40}{22} \times 100 = \frac{40}{22} = \frac{20}{11} = 1\frac{9}{11}\%$

Hence, the profit or gain is $1\frac{9}{11}\%$.

8. Let CP of the horse be ₹ x

Then gain percentage = 5% [Given]

Gain = 5% of x = $\frac{5}{100} \times x = \frac{x}{20}$

SP = CP + Gain = $x + \frac{x}{20} = \frac{21x}{20}$

Loss = 2% of x = $\frac{2}{100} \times x = \frac{x}{50}$

$\therefore$ SP = CP – Gain = $x - \frac{x}{50} = \frac{49\,x}{50}$

$\therefore$ Difference between two SPs = $\frac{21\,x}{20} - \frac{49x}{50} = \frac{105x}{100} - \frac{98x}{100}$ = ₹ $\frac{7x}{100}$

If difference in SP is ₹ $\frac{7x}{100}$, then CP = x

If difference in SP is ₹ 490, then CP = ₹ $\frac{\cancel{x} \times \cancel{490}^{70} \times 100}{\cancel{7}\ \cancel{x}}$ = ₹ 7,000.

Hence, the cost price of the horse is ₹ 7,000.

9. Given : SP of onions = ₹ 1232 and profit = 12%

(a) Let CP of onions be ₹ x.

Then profit = 12% of x = $\frac{12}{100} \times x = \frac{3x}{25}$

$\therefore$ SP of onions = ₹ $\left(x+\frac{3x}{25}\right)$ = ₹ $\frac{28x}{25}$

$\therefore \frac{28x}{25}$ = ₹ 1232 $\quad \Rightarrow x$ = ₹ $\frac{1232 \times 25}{28}$ = ₹ 1100.

Hence, the cost price of the onions is Rs. 1100.

(b) Given SP of potatoes = ₹ 1232 and loss = 12%

Let CP of potatoes be ₹ x.

Then loss = 12% of x = $\frac{3x}{25}$

SP of potatoes $= x - \frac{3x}{25} = \frac{22x}{25}$

$\therefore \frac{22x}{25}$ = ₹ 1232. $\quad \Rightarrow x = \frac{1232 \times 25}{22}$ = ₹ (56 × 25) = ₹ 1400.

Hence, the cost price of potatoes is ₹ 1400.

(c) Total CP of onions and potatoes = ₹ 1100 + ₹ 1400 = ₹ 2500.

Total SP of onions and potatoes = ₹ 1232 + ₹ 1232 = ₹ 2464.

As CP > SP, so there is a loss in the transaction.

Loss = CP – SP = ₹ 2500 – ₹ 2464 = ₹ 36

$\therefore$ Loss% $= \frac{\text{Loss}}{\text{CP}} \times 100 = \frac{36}{\cancel{2500}} \times \cancel{100} = \frac{36}{25} = 1\frac{11}{25}\%$

Hence, the loss percentage in the whole transaction is $1\frac{11}{25}\%$.

10. Given SP of a cooler = ₹ 24,225 and successive discounts 15% and 5%.

Let MP of the cooler be ₹ x.

Then after 15% discount price of the cooler = x – 15% of x $= x - \frac{15x}{100} = \frac{85x}{100}$

Again after 5% discount price of the cooler $= \frac{85x}{100} \times \left(1 - \frac{5}{100}\right) = \frac{85x}{100} \times \frac{95}{100} = 24225.$

$x = \frac{24225 \times 100 \times 100}{85 \times 95} = 30{,}000.$

Hence, the marked price of the cooler is ₹ 30,000.

11. Let MP of the memory card be ₹ x.

Then rebate = 10% of x $= \frac{10}{100} \times x = \frac{x}{10}$

Price of the memory card after rebate $= x - \frac{x}{10} = \frac{9x}{10}.$

Sales tax = 5% of $\frac{9x}{10} = \frac{\cancel{5}^{1}}{\cancel{100}_{20}} \times \frac{9x}{10} = \frac{9x}{200}.$

Amount including sales tax $= \frac{9x}{10} + \frac{9x}{200} = \frac{180x + 9x}{200} = \frac{189x}{200}.$

According to the question,

$\frac{189x}{200}$ = ₹ 945

$\Rightarrow$ 189 x = ₹ 945 × 200

$\Rightarrow$ x = ₹ $\dfrac{945 \times 200}{189}$ = ₹ 1000

Hence, the marked price of the memory card is ₹ 1000.

12. Let MP of the jeans be ₹ x.

The MP of the tops will be ₹ (980 – x)

VAT on the jeans = 10% of x = ₹ $\dfrac{10}{100} \times x$ = ₹ $\dfrac{x}{10}$.

VAT on the tops = 5% of ₹ (980 – x) = ₹ $\dfrac{5}{100} \times (980 - x)$ = ₹ $\dfrac{1}{20} \times (980 - x)$

According to the question,

$\dfrac{x}{10} + \dfrac{980 - x}{20}$ = ₹ 94 $\quad\Rightarrow\quad \dfrac{2x + 980 - x}{20}$ = ₹ 94

$\Rightarrow$ x + 980 = ₹ (94 × 20) = ₹ 1800 $\quad\Rightarrow$ x = 1880 – 980 = ₹ 900.

Hence, the marked price of the jeans is ₹ 900 and that of tops is ₹ (980 – 900) = ₹ 80.

Multiple Choice Questions

1. Gain = SP – CP = ₹ 75 – ₹ 60 = ₹ 15.

Gain% = $\dfrac{\text{Gain}}{\text{CP}} \times 100 = \dfrac{\cancel{15} \times \cancel{100}^{25}}{\cancel{60}_{4}}$ = 25%

Hence, the correct answer is (b).

2. Let CP of the item be ₹ x.

Then gain = 8% of x = $\dfrac{8}{100} \times x = \dfrac{2x}{25}$

SP of the item = $x + \dfrac{2x}{25} = \dfrac{27x}{25}$

MP of the item = 35% of CP + x = $\dfrac{35}{100} \times x + x = \dfrac{7x}{20} + x = \dfrac{27x}{20}$

Discount = MP – SP

$= \dfrac{27x}{20} - \dfrac{27x}{25} = \dfrac{135x - 108x}{100} = \dfrac{27x}{100}$

Discount% = $\dfrac{\text{Discount}}{\text{MP}} \times 100 = \left(\dfrac{27x}{100} \div \dfrac{27x}{20}\right)100 = \left(\dfrac{\cancel{27x}}{100} \times \dfrac{20}{\cancel{27x}}\right)100 = \dfrac{20 \times \cancel{100}}{\cancel{100}}$ = 20%.

Hence, the correct option is 20%.

3. Let the CP of the dress be ₹ x.

Then gain = 10% of x = $\dfrac{10}{100} \times x = \dfrac{x}{10}$

SP of the dress = CP + gain = $x + \dfrac{x}{10} = \dfrac{11x}{100}$

According to the question,

$\dfrac{11x}{10}$ = ₹ 528 $\quad\Rightarrow$ x = ₹ $\dfrac{528 \times 10}{11}$ = ₹ 480.

Hence, the correct option is (d).

4. Let CP of the mobile be ₹ x.

Then VAT = 12% of x = $\frac{12}{100} \times x = \frac{3x}{25}$

Price including VAT = $x + \frac{3x}{25} = \frac{28x}{25}$

According to the question,

$\frac{28x}{25}$ = ₹ 4480 ⇒ x = ₹ $\frac{4480 \times 25}{28}$ = ₹ 4000.

Hence, the correct option is (d).

5. Let CP of the article be ₹ x.

Then its MP = $x + 10\% \, x = x + \frac{10}{100} \times x = \frac{11x}{10}$.

Discount = 10% of MP = $\frac{\cancel{10}}{100} \times \frac{11x}{\cancel{10}} = \frac{11x}{100}$

SP = MP – discount = $\frac{11x}{10} - \frac{11x}{100} = \frac{110x - 11x}{100} = \frac{99x}{100}$

As SP < CP, so there is a loss.

Loss = CP – SP = $x - \frac{99x}{100} = \frac{100x - 99x}{100} = \frac{x}{100}$

Loss% = $\frac{\text{Loss}}{\text{CP}} \times 100 = \left(\frac{x}{100 \times x} \times 100\right)\% = 1\%$

Hence, the correct option is (b).

6. Ratio 3 : 5 as percentage = $\frac{3}{\cancel{5}} \times \cancel{100}^{20} = 2 \times 20 = 60\%$

Hence, the correct option is (b).

7. $\frac{1}{3}\%$ of 9000 m = $\frac{1}{3} \times \frac{1}{1\cancel{00}} \times 90\cancel{00}$ m = $\frac{90}{3}$ m = 30 m. [1 km = 1000m]

Hence, the correct option is (d).

8. Required ratio = 30 : 300 = 1 : 10. [1 ₹ = 100 p]

Hence, the correct option is (a).

9. 18% of 500 = x × 0.3

⇒ $\frac{18}{\cancel{100}} \times \cancel{500}^{5} = 0.3\,x$

⇒ $90 = 0.3\,x = x = \frac{90}{0.3} = \frac{\cancel{900}^{300}}{\cancel{3}} = 300.$

Hence, the correct option is (a).

10. Discount = MP – SP = ₹ 8000 – ₹ 7750 = ₹ 250

Discount% = $\frac{\text{Discount}}{\text{MP}} \times 100 = \frac{250}{80\cancel{00}} \times 1\cancel{00} = \frac{25\cancel{0}}{8\cancel{0}} = \frac{25}{8} = 3\frac{1}{8}\%$.

Hence, the correct option is (a).

Mental Maths

A. See the Answers given in the book.

B. See the Answers given in the book.

Higher Order Thinking Skills (HOTS)

1. Let the income of Rahul be ₹ x.

   Then Sahil's income = x + 10% of x = $x + \frac{10x}{100} = x + \frac{x}{10} = \frac{11x}{10}$

   Difference between their incomes = $\frac{11x}{10} - x = \frac{11x - 10x}{10} = \frac{x}{10}$.

   Percentage of Rahul's income of Sahil's income = $\left(\frac{x}{10} \div \frac{11x}{10}\right) \times 100\%$

   $= \frac{\cancel{x} \times \cancel{10}}{\cancel{10} \times 11\cancel{x}} \times 100\% = 9\frac{1}{11}\%$.

   Hence, Rahul's income is $9\frac{1}{11}\%$ less than that of Sahil.

2. Let CP of the pen drive be ₹ x.

   Then profit = 12% of x = ₹ $\frac{12}{100} \times x$ = ₹ $\frac{3x}{25}$.

   SP of the pen drive = $x + \frac{3x}{25}$ = ₹ $\frac{28x}{25}$.

   ∴ CP of the pen drive for Sunita is ₹ $\frac{28x}{25}$.

   Loss for Sunita = $\frac{28x}{25} = \frac{\cancel{5}}{\cancel{100}_{20}} \times \frac{28x}{25} = \frac{28x}{500}$.

   ∴ SP of the pen drive = ₹ $\frac{28x}{25}$ − ₹ $\frac{28x}{500}$ = ₹ $\frac{560x - 28x}{500} = \frac{532x}{500}$

   According to the question,

   $\frac{532x}{500}$ = ₹ 1596 ⇒ x = ₹ $\frac{1596 \times 500}{532}$ ₹ (3 × 500) = ₹ 1500

   Hence, Amit paid ₹ 1500 for the pen drive.

3. CP of 1 watch = ₹ 850

   Loss = 5% of ₹ 850 = $\frac{5}{100}$ × ₹ 850 = ₹ 42.50

   SP of this watch = ₹ 850 – ₹ 42.50 = ₹ 807.50.

   CP of 4 watches = ₹ 850 × 4 = ₹ 3400.

   Given gain = 10%

   ∴ SP of 4 watches = ₹ 3400 + 10% of ₹ 3400 = ₹ 3400 + ₹ 340 = ₹ 3740.

   ∴ SP of 3 watches = ₹ 3740 – ₹ 807.50 = ₹ 2932.50

   ∴ SP of 1 watch = ₹ 2932.50 ÷ 3 = ₹ 977.50.

   Hence, the price of cash of remaining three watches should be ₹ 977.50.

10 Compound Interest

Exercise 10.1

1. Given : Principal = ₹ 5000, rate = 4% and time = 2 years.

   Simple interest of first year $= \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100} = ₹\ \frac{50\cancel{00} \times 4 \times 1}{1\cancel{00}}$ = ₹ 50 × 4 = ₹ 200.

   Amount = ₹ 5000 + ₹ 200 = ₹ 5200.

   Principal for the second year = ₹ 5200, rate = 4% and time = 1 year.

   ∴ Simple interest for the second year $= \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100} = ₹\ \frac{52\cancel{00} \times 4 \times 1}{1\cancel{00}}$ = ₹ 208.

   ∴ Amount = Principal + Interest = ₹ 5200 + ₹ 208 = ₹ 5408.

   Hence, the compound interest for 2 years = ₹ 5408 – ₹ 5000 = ₹ 408.

2. Similar work to be done as Q.1.

3. Given : Principal (P) = ₹ 14000, rate (R) = 8% per annum and time (T) = 2 years.

   ∴ SI $= \frac{P \times R \times T}{100} = ₹\ \frac{140\cancel{00} \times 8 \times 2}{1\cancel{00}}$ = ₹ 140 × 16 = ₹ 2240.

   For compound interest:

   Principal for 1st year = ₹ 14000, rate (R) = 8% and time (T) = 1 year

   ∴ Interest $= \frac{P \times R \times T}{100} = ₹\ \frac{140\cancel{00} \times 8 \times 1}{1\cancel{00}}$ = ₹ 1120

   Principal for 2nd year = ₹ 14000 + ₹ 1120 = ₹ 15120.

   R = 8% and T = 1 year.

   ∴ Interest $= \frac{P \times R \times T}{100} = ₹\ \frac{15120 \times 8 \times 1}{100}$ = ₹ 1209.60

   Amount = Principal + Interest = ₹ 15120 + ₹ 1209.60 = ₹ 16329.60

   Compound interest = ₹ 16329.60 – ₹ 14000 = ₹ 2329.60

   ∴ Difference between compound interest and simple interest = ₹ 2329.60 – ₹ 2240 = ₹ 89.60.

4. Principal for the first year = ₹ 1500, R = 4% per annum and T = 1 year

   ∴ Interest for the first year $= \frac{P \times R \times T}{100} = ₹\ \frac{15\cancel{00} \times 4 \times 1}{1\cancel{00}}$ = ₹ 60.

   ∴ Amount = P + SI = ₹ 1500 + ₹ 60 = ₹ 1560.

   Principal for second year = ₹ 1560, R = 4% and T = 1 year.

   ∴ Simple interest $= \frac{P \times R \times T}{100} = ₹\ \frac{1560 \times 4 \times 1}{100}$ = ₹ 62.40.

   Amount = P + SI = ₹ 1560 + ₹ 62.40 = ₹ 1622.40.

   ∴ Principal for the third year = ₹ 1622.40, R = 4% and T = 1 year.

   ∴ SI $= \frac{P \times R \times T}{100} = ₹\ \frac{1622.40 \times 4 \times 1}{100}$ = ₹ 64.90

   Amount = P + SI = ₹ 1622.40 + ₹ 64.90 = ₹ 168730.

   ∴ Compound interest = ₹ 1687.30 – ₹ 1500 = ₹ 187.30

   Hence, the compound interest earned by Shubham is ₹ 187.30.

5. Similar work to be done as Q. 3.

6. Given : P = ₹ 20000, R = 16% per annum and T = 2 years.

$\therefore$ Simple interest $= \frac{P \times R \times T}{100} = ₹\ \frac{20,0\cancel{00} \times 16 \times 1}{1\cancel{00}}$ = ₹ 6400.

For compound interest:

For the first year, P = ₹ 20,000, R = 16% and T = 1 year.

$\therefore$ SI = ₹ $\frac{20,0\cancel{00} \times 16 \times 1}{1\cancel{00}}$ = ₹ 200 × 16 = ₹ 3200.

$\therefore$ Amount = ₹ 20,000 + ₹ 3200 = ₹ 23,200.

For the second year, P = ₹ 23,200, R = 16% and T = 1 year

$\therefore$ SI = ₹ $\frac{232\cancel{00} \times 16 \times 1}{1\cancel{00}}$ = ₹ 3712.

$\therefore$ Amount = ₹ 23,200 + ₹ 3712 = ₹ 26,912.

Compound Interest = ₹ (26912 – 20000) = ₹ 6912.

$\therefore$ Difference between the compound interest and simple interest = ₹ 6912 – ₹ 6400 = ₹ 512.

Hence, the required gain for Mrs. Shashi is ₹ 512.

7. Given : P = ₹ 4500, $T = 1\frac{1}{2} = \frac{3}{2}$ years = 3 half years and R = 10% per annum = 5% per half year.

For the first half year : P = ₹ 4500, T = 1 and R = 5%

$\therefore$ S I $= \frac{P \times R \times T}{100}$ = ₹ $\frac{45\cancel{00} \times 5 \times 1}{1\cancel{00}}$ = ₹ 45 × 5 = ₹ 225.

Amount = P + SI = ₹ 4500 + ₹ 225 = ₹ 4725.

For the second half year : P = ₹ 4725, R = 5% and T = 1

$\therefore$ SI $= \frac{P \times R \times T}{100}$ = ₹ $\frac{4725 \times 5 \times 1}{100}$ = ₹ 236.25

Amount = P + SI = ₹ 4725 + ₹ 236.25 = ₹ 4961.25

For third year : P = ₹ 4961.25, R = 5% and T = 1

$\therefore$ SI = $\frac{P \times R \times T}{100}$ = ₹ $\frac{4961.25 \times 5}{100}$ = ₹ 248.06

Amount = P + SI = ₹ 4961.25 + ₹ 248.06 = ₹ 5209.31.

Compound interest = ₹ 5209.31 – ₹ 4500 = ₹ 709.31.

Thus, the amount and compound interest are respectively ₹ 5209.31 and ₹ 709.31.

8. Given : P = ₹ 10,500, R = 2.5% and T = 3 years.

For First year : P = ₹ 10500, R = 2.5% and T = 1 year

$\therefore$ SI $= \frac{P \times R \times T}{100}$ = ₹ $\frac{10500 \times 2.5 \times 1}{100}$ = ₹ 262.50.

$\therefore$ Amount = ₹ 10500 + ₹ 262.50 = ₹ 10762.50

For second year : P = ₹ 10762.50, R = 2.5% and T = 1 year.

$\therefore$ SI $= \frac{P \times R \times T}{100}$ = ₹ $\frac{10762.50 \times 2.5 \times 1}{100}$ = ₹ 269.06.

Amount = P + SI = ₹ 10762.50 + ₹ 269.06 = ₹ 1103156

For third year : P = ₹ 11031.56, R = 2.5% and T = 1 year

$\therefore$ SI $= \frac{P \times R \times T}{100} = ₹\ \frac{11031.56 \times 2.5 \times 1}{100} = ₹\ 275.79.$

Amount = ₹ 11031.56 + ₹ 275.79 = ₹ 11307.35

$\therefore$ Compound interest = Amount – P = ₹ 11307.35 – ₹ 10500 = ₹ 807.35

Hence, the required compound interest is ₹ 807.35.

9. Given : P = ₹ 7500, R = 6% per annum and T = 1 year.

As the interest is compounded half yearly, so R = 3% per half year and T = 2 half years.

For first half year : P = ₹ 7500, R = 3% and T = 1

$\therefore$ SI $= \frac{P \times R \times T}{100} = ₹\ \frac{7500 \times 3 \times 1}{100} = ₹\ 225.$

$\therefore$ Amount = P + SI = ₹ 7500 + ₹ 225 = ₹ 7725.

For second half year : P = ₹ 7725, R = 3% and T = 1

$\therefore$ SI $= \frac{P \times R \times T}{100} = ₹\ \frac{7725 \times 3 \times 1}{100} = ₹\ 231.75$

Amount = P + SI = ₹ 7725 + ₹ 231.75 = ₹ 7956.75

$\therefore$ Compound interest = ₹ 7956.75 – ₹ 7500 = ₹ 456.75.

Hence, Aditya will receive ₹ 456.75 as compound interest.

Exercise 10.2

1. (a) Given : P = ₹ 8000, R = 9% per annum and n = 3 years

$\therefore$ Amount $= P\left(1 + \frac{R}{100}\right)^n = ₹\ 8000\left(1 + \frac{9}{100}\right)^3$

$= ₹\ 8000\left(\frac{109}{100}\right)^3 = ₹\ 8000 \times \frac{109}{100} \times \frac{109}{100} \times \frac{109}{100} = ₹\ 10360.23.$

$\therefore$ Compound interest = ₹ 10360.23 – 8000 = ₹ 2360.23.

(b) Given : P = ₹ 12000, R = 12% per annum and n = 18 months =

As the interest is compounded half-yearly, so R = 6% and n = 3 years.

$\therefore$ Amount $= P\left(1 + \frac{R}{100}\right)^n = ₹\ 12000\left(1 + \frac{6}{100}\right)^3 = ₹\ 12000\left(\frac{106}{100}\right)^3$

$= ₹\ 12000 \times \frac{106}{100} \times \frac{106}{100} \times \frac{106}{100} = ₹\ 12000 \times 1.06 \times 1.06 \times 1.06 = 14292.19$

$\therefore$ Compound interest = ₹ 14292.19 – ₹ 12000 = ₹ 2292.19

(c) Given : P = ₹ 7500, R = 12% and n = 9 months

As the interest is compounded quarterly, so n = 9 month $= \frac{9}{12}$ Years $= \frac{3}{4}$ Years = 3 quarter

R = 12% ÷ 4 = 3% quarterly.

$\therefore$ Amount $= P\left(1 + \frac{R}{100}\right)^n = ₹\ 7500\left(1 + \frac{3}{100}\right)^3 = ₹\ 7500\left(\frac{103}{100}\right)^3 = ₹\ 7500 \times \frac{103}{100} \times \frac{103}{100} \times \frac{103}{100}$

= ₹ 7500 × 1.03 × 1.03 × 1.03 = ₹ 8195.45

$\therefore$ Compound interest = ₹ 8195.45 – 7500 = ₹ 695.45

2. Given : P = ₹ 10240, R = $12\frac{1}{2} = \frac{25}{2}$% and n = 2 years

$\therefore$ Amount = $P\left(1+\frac{R}{100}\right)^n$ = ₹ $10240\left(1+\frac{25}{200}\right)^2$ = ₹ $10240\left(\frac{225}{200}\right)^2$ = ₹ $10240\left(\frac{9}{8}\right)^2$

= ₹ $10240 \times \frac{9}{8} \times \frac{9}{8}$ = ₹ 12960.

$\therefore$ Compound interest = A – P = ₹ 12960 – 10240 = ₹ 2720.

Hence, the compound interest is ₹ 2720.

3. Given : P = ₹ 31250, n = $2\frac{1}{2}$ years = $\frac{5}{2}$ years and R = 6% per annum

Amount = $P\left(1+\frac{R}{100}\right)^n$ = ₹ $31250\left(1+\frac{6}{100}\right)^{\frac{5}{2}}$

= ₹ $31250\left(\frac{106}{100}\right)^2\left(1+\frac{3}{100}\right)$ $\left[\text{For } \frac{1}{2} \text{ yr, } r = \frac{6}{2} = 3\%\right]$

= ₹ $31250 \times \frac{106}{100} \times \frac{106}{100} \times \frac{103}{100}$ = ₹ 31250 × 1.06 × 1.06 × 1.03 = ₹ 36165.88.

$\therefore$ Compound interest = A – P = ₹ 36165.88 – ₹ 31250 = ₹ 4915.88.

4. Given : P = ₹ 6000, R = 10% per annum and n = 2 years 4 months.

Amount for the first 2 years :

A = $P\left(1+\frac{R}{100}\right)^n$ = ₹ $6000\left(1+\frac{10}{100}\right)^2$ = ₹ $6000\left(\frac{11}{10}\right)^2$ = ₹ $60\cancel{00} \times \frac{11}{\cancel{10}} \times \frac{11}{\cancel{10}}$

= ₹ 60 × 11 × 11 = ₹ 7260.

Compound interest for 4 months = $\frac{4}{12} = \frac{1}{3}$ years, P = ₹ 7260 and R = 10%.

$\therefore$ I = $\frac{P \times R \times T}{100}$ = ₹ $\frac{7260 \times \cancel{10}^{1} \times 1}{\cancel{100}_{10} \times 3}$ = ₹ 242.

Total amount = ₹ 7260 + ₹ 242 = ₹ 7502.

$\therefore$ Compound interest = ₹ 7502 – ₹ 6000 = ₹ 1502.

Hence, the compound interest is ₹ 1502.

5. Given : R = 5%, T = 3 years and SI = ₹ 2400.

$\therefore$ SI = $\frac{P \times R \times T}{100}$

$\Rightarrow$ ₹ 2400 = $\frac{P \times 5 \times 3}{100}$

$\Rightarrow$ ₹ 2400 × 100 = 15 P

$\Rightarrow$ P = ₹ $\frac{2,40,000}{15}$ = ₹ 16000.

Now, amount = $P\left(1+\frac{R}{100}\right)^n$ = ₹ $16000\left(1+\frac{5}{100}\right)^3$

= ₹ $16000\left(\frac{105}{100}\right)^3$ = ₹ $16000\left(\frac{21}{20}\right)^3$ = ₹ $16000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}$

$= ₹\ \cancel{16000}^{2} \times \frac{9261}{\cancel{8000}_{1}} = ₹\ 9261 \times 2 = ₹\ 18522$

∴ Compound interest = ₹ 18522 – ₹ 16000 = ₹ 2522.

Hence, the compound interest is ₹ 2522.

6. Given : P = ₹ 36000, R = 10% and n = 2 years

$\therefore \text{Amount} = P\left(1+\frac{R}{100}\right)^{n} = ₹\ 36000\left(1+\frac{10}{100}\right)^{2} = ₹\ 36000\left(\frac{11}{10}\right)^{2}$

$= ₹\ 36000 \times \frac{11}{10} \times \frac{11}{10} = ₹\ 360 \times 121 = ₹\ 43560.$

Hence, Suraj will pay ₹ 43560 after 2 years.

7. Given : P = ₹ 65536, R = 12.5% and n = 2 years.

When interest is compounded annually :

$\text{Amount} = P\left(1+\frac{R}{100}\right)^{n} = ₹\ 65,536\left(1+\frac{12.5}{100}\right)^{2} = ₹\ 65,536\left(1+\frac{125}{1000}\right)^{2} = ₹\ 65,536\left(1+\frac{1}{8}\right)^{2}$

$= ₹\ 65,536\left(\frac{9}{8}\right)^{2} = ₹\ 65,536 \times \frac{9}{8} \times \frac{9}{8} = ₹\ 82,944$

Compound interest = ₹ 82,944 – ₹ 65536 = ₹ 17408.

When interest is compounded half yearly: P = ₹ 65,536, R = 12.5% ÷ 2 = 6.25% and n = 4 half years.

$\therefore \text{Amount} = P\left(1+\frac{R}{100}\right)^{n} = ₹\ 65,536\left(1+\frac{6.25}{100}\right)^{4} = ₹\ 65,536\left(1+\frac{625}{10000}\right)^{4}$

$= ₹\ 65,536\left(1+\frac{1}{16}\right)^{4} = ₹\ 65,536\left(\frac{17}{16}\right)^{4} = ₹\ 65,536 \times \frac{17}{16} \times \frac{17}{16} \times \frac{17}{16} \times \frac{17}{16} = ₹\ 83,531.$

∴ Compound interest = A – P = ₹ 83,531 – 65,536 = ₹ 17,985.

Difference between these two compound interests = ₹ 17,985 – ₹ 17,408 = ₹ 577.

Hence, Shubham will earn ₹ 577 more if the interest is compounded half yearly.

8. Given : P = ₹ 2,048 – $R = 12\frac{1}{2} = \frac{25}{2}\%$ and n = 18 months.

As the interest is compounded half-yearly, so $R = \frac{25}{4}\%$ and $n = \frac{18}{12} = \frac{3}{2} = 3$ half years.

$\therefore \text{Amount} = P\left(1+\frac{R}{100}\right)^{n} = ₹\ 2048\left(1+\frac{25}{400}\right)^{3} = ₹\ 2048\left(1+\frac{1}{16}\right)^{3}$

$= ₹\ 2048\left(\frac{17}{16}\right)^{3} = ₹\ 2048 \times \frac{17}{16} \times \frac{17}{16} \times \frac{17}{16} = ₹\ 2456.50.$

Hence, the amount is = ₹ 2456.50.

9. Given : P = ₹ 24,000, R = 20% and n = 1 year

As the interest is compounded quarterly.

$\therefore R = \frac{20}{4}\% = 5\%$ and n = 1 year = 4 quarters.

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^n = ₹\ 24,000\left(1+\frac{5}{100}\right)^4 = ₹\ 24,000\left(1+\frac{1}{20}\right)^4$$

$$= ₹\ 24,000\left(\frac{21}{20}\right)^4 = ₹\ 24,000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} = ₹\ 29172.15$$

∴ Compound interest = ₹ 29172.15 – ₹ 24000 = ₹ 5172.15

Hence, the compound interest is ₹ 5172.15

10. Given : P = 4125, n = 3 years and R_1 = 4%, R_2 = 5% and R_3 = 10%

$$\therefore \text{Amount} = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right) = ₹\ 4125\left(1+\frac{\cancel{4}^{1}}{\cancel{100}_{25}}\right)\left(1+\frac{\cancel{5}^{1}}{\cancel{100}_{20}}\right)\left(1+\frac{\cancel{10}^{1}}{\cancel{100}_{10}}\right)$$

$$= ₹\ 4125 \times \frac{26}{25} \times \frac{21}{20} \times \frac{11}{10} = ₹\ 4954.95$$

∴ Compound interest = Amount – Principal = ₹ 4954.95 – ₹ 4125 = ₹ 829.95

Hence, the compound interest is ₹ 829.95.

11. Given : P = ₹ 16000, n = 2 years, R_1 = 4% and R_2 = 8%

$$\therefore \text{Amount} = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right) = ₹\ 16,000\left(1+\frac{4}{100}\right)\left(1+\frac{8}{100}\right) = ₹\ 16,000\left(\frac{26}{25}\right)\left(\frac{27}{25}\right)$$

$$= ₹\ \frac{16,000 \times 26 \times 27}{25 \times 25} = ₹\ 17,971.20$$

∴ Compound interest = ₹ 17971.20 – ₹ 16000 = ₹ 1971.20

Exercise 10.3

1. Given : Population of a city, P = 184000, rate, R = 5% per annum.

(a) Here, time, n = 2015 – 2012 = 3 years

$$\therefore \text{Population before 3 years, } A = P\left(1-\frac{R}{100}\right)^n = ₹\ 1,84,000\left(1-\frac{5}{100}\right)^3$$

$$= ₹\ 1,84,000\left(\frac{19}{20}\right)^3 = ₹\ 1,84,000 \times \frac{19}{20} \times \frac{19}{20} \times \frac{19}{20} = ₹\ 1,57,757.$$

Hence, the population of the city is 2012 was ₹ 1,57,757.

(b) Here, n = 2017 – 2015 = 2 years.

$$\therefore \text{Population is 2017, } A = P\left(1+\frac{R}{100}\right)^n = ₹\ 1,84,000\left(1+\frac{5}{100}\right)^2$$

$$= ₹\ 1,84,000\left(\frac{21}{20}\right)^2 = ₹\ 1,84,000 \times \frac{21}{20} \times \frac{21}{20} = ₹\ 2,02,860.$$

Hence, the population of the city in 2017 was ₹ 2,02,860.

2. Given : Population of the village, P = 64000, R = 2.5% per annum and n = 3 years.

∴ Population of the village at the end of 3 years,

$$A = P\left(1+\frac{R}{100}\right)^n = ₹\ 64,000\left(1+\frac{2.5}{100}\right)^3 = ₹\ 64,000\left(1+\frac{25}{1000}\right)^3 = ₹\ 64,000\left(1+\frac{1}{40}\right)^3$$

$$= ₹\ 64,000\left(\frac{41}{40}\right)^3 = ₹\ 64,000 \times \frac{41}{40} \times \frac{41}{40} \times \frac{41}{40} = ₹\ 68,921.$$

Hence, the population of the village at the end of 3 years is ₹ 68,921.

3. Given : Value of a residential flat, P = ₹ 18,00,000, R = 10% per annum and n = 3.

$\therefore$ Value of the flat after 3 years, $A = P\left(1+\frac{R}{100}\right)^n = ₹\ 18,00,000\left(1+\frac{10}{100}\right)^3$

$$= ₹\ 18,00,000\left(\frac{11}{10}\right)^3 = ₹\ 18,00,000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10} = ₹\ 23,95,800.$$

Hence, the value of the flat 3 years after construction is ₹ 23,95,800.

4. Given : P = 125000, R = 4% per annum and n = 2 years.

$$\therefore A = P\left(1-\frac{R}{100}\right)^n = ₹\ 1,25,000\left(1-\frac{4}{100}\right)^2 = ₹\ 1,25,000\left(\frac{24}{25}\right)^2 = ₹\ 1,15,200.$$

Hence, the present population of the city is = ₹ 1,15,200.

5. Given : P = ₹ 6,40,000, R = 10% per annum and n = 2 years and 3 years.

(a) Here, n = 2 years.

$$\therefore A = P\left(1-\frac{R}{100}\right)^n = ₹\ 6,40,000\left(1-\frac{10}{100}\right)^2 = ₹\ 6,40,000\left(1-\frac{1}{10}\right)^2 \quad [\because \text{Value is depreciated}]$$

$$= ₹\ 6,40,000\left(\frac{9}{10}\right)^2 = ₹\ 6,40,000 \times \frac{9}{10} \times \frac{9}{10} = ₹\ 5,18,400.$$

Hence, the value of the car after 2 years will be ₹ 5,18,400.

(b) Here, n = 3 years.

$$\therefore A = P\left(1-\frac{10}{100}\right)^3 = ₹\ 6,40,000\left(1-\frac{1}{10}\right)^3 = ₹\ 6,40,000 \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} = ₹\ 4,66,560.$$

Hence, the value of the car after 3 years will be ₹ 4,66,560.

6. Given : A = 40000, R = 5% per annum and n = 3 years.

$$\therefore A = P\left(1+\frac{R}{100}\right)^n = 40,000\left(1+\frac{5}{100}\right)^3 = 40,000\left(\frac{21}{20}\right)^3 = 40,000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} = 46,305.$$

Hence, the production of motorcycles after 3 years is 46,305.

7. Given : P = 1,44,000, R = 5% per annum and n = 3 years.

$$\therefore A = P\left(1+\frac{R}{100}\right)^n = 1,44,000\left(1+\frac{5}{100}\right)^3 = 1,44,000\left(\frac{21}{20}\right)^3$$

$$= 1,44,000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} = 1,66,698.$$

Hence, the number of inhabitants after 3 years will be 1,66,698.

8. Given : P = ₹ 85,000, R_1 = 20% and R_2 = 10% per annum

$\therefore$ Value of the bike after 2 years,

$$A = P\left(1-\frac{20}{100}\right)\left(1-\frac{10}{100}\right) = ₹\ 85{,}000\left(\frac{4}{5}\right)\left(\frac{9}{10}\right) = ₹\ 85{,}000 \times \frac{4}{5} \times \frac{9}{10} = ₹\ 61200.$$

Hence, Suresh will get ₹ 61200 for the bike.

9. Given : P = 400,000, $R_1 = 2.5\%$, $R_2 = 5\%$ and $R_3 = 2\%$ per annum.

∴ Present population of the district,

$$A = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right) = 4{,}00{,}000\left(1+\frac{25}{100}\right)\left(1+\frac{5}{100}\right)\left(1+\frac{2}{100}\right)$$

$$= 4{,}00{,}000\left(1+\frac{1}{40}\right)\left(1+\frac{1}{20}\right)\left(1+\frac{1}{50}\right)$$

$$= 4{,}00{,}000 \times \frac{41}{40} \times \frac{21}{20} \times \frac{51}{50} = 4{,}39{,}110.$$

Hence, the present population of the district is 4,39,110.

10. Given : P = 240000, $R_1 = 15\%$, $R_2 = 6\%$ and $R_3 = 5\%$ per annum.

∴ Population of the town after 3 years,

$$A = P\left(1-\frac{15}{100}\right)\left(1+\frac{6}{100}\right)\left(1+\frac{5}{100}\right) = 2{,}40{,}000\left(\frac{17}{20}\right)\left(\frac{53}{50}\right)\left(\frac{21}{20}\right)$$

$$= 2{,}40{,}000 \times \frac{17}{20} \times \frac{53}{50} \times \frac{21}{20} = 2{,}27{,}052.$$

Hence, the population of the town at the end of the third year is 2,27,052.

11. Given : P = 4,08,000, R = 2.5% per annum and n = 2 hours.

∴ Number of bacteria after 2 hours.

$$A = P\left(1+\frac{R}{100}\right)^n = 4{,}08{,}000\left(1+\frac{2.5}{100}\right)^2 = 4{,}08{,}000\left(1+\frac{1}{40}\right)^2 = 4{,}08{,}000 \times \frac{41}{40} \times \frac{41}{40} = 4{,}28{,}655.$$

Hence, the number of bacteria at the end of two hours is 4,28,655.

Exercise 10.4

1. Given : Compound interest, CI = ₹ 2,023, $R = 12\frac{1}{2}\% = \frac{25}{2}\%$ = 2 years.

$$\therefore CI = P\left[\left(1+\frac{R}{100}\right)^n - 1\right]$$

$$\Rightarrow ₹\ 2023 = P\left[\left(1+\frac{25}{200}\right)^2 - 1\right]$$

$$\Rightarrow ₹\ 2023 = P\left[\frac{9}{8}-1\right]$$

$$\Rightarrow ₹\ 2023 = P \times \frac{1}{8}$$

∴ P = ₹ 2023 × 8 = ₹ 16,184.

Hence, the required principal is ₹ 16,184.

2. Given : A = ₹ 1681, P = ₹ 1600 and R = 5% per annum.

As the interest is compounded half-yearly, so $R = \frac{5}{2} = 2.5\%$ and time = 2n.

$\therefore$ Amount = $P\left(1+\frac{R}{100}\right)^n$

$\Rightarrow$ ₹ 1681 = ₹ $1600\left(1+\frac{2.5}{100}\right)^{2n}$

$\Rightarrow$ ₹ $\frac{1681}{1600}$ = ₹ $\left(1+\frac{1}{40}\right)^{2n}$

$\Rightarrow \frac{1681}{1600} = \left(\frac{41}{40}\right)^{2n}$

$\Rightarrow \frac{(41)^2}{(40)^2} = \left(\frac{41}{40}\right)^{2n}$

$\Rightarrow 2 = 2n$ [Bases are same.]

$\therefore n = 1$

Hence, the required time is 1 year.

3. Given : A = ₹ 3,63,000, P = ₹ 3,00,000 and n = 2 years.

$\therefore A = P\left(1+\frac{R}{100}\right)^n$

$\Rightarrow$ ₹ 3,63,000 = ₹ 3,00,000 $\left(1+\frac{R}{100}\right)^2$

$\Rightarrow \frac{3,63,000}{3,00,000} = \left(1+\frac{R}{100}\right)^2$

$\Rightarrow \frac{363}{300} = \left(1+\frac{R}{100}\right)^2$

$\Rightarrow \frac{121}{100} = \left(1+\frac{R}{100}\right)^2$

$\Rightarrow \left(\frac{11}{10}\right)^2 = \left(1+\frac{R}{100}\right)^2$

$\Rightarrow \frac{11}{10} = 1+\frac{R}{100}$ [Powers are same.]

$\Rightarrow \frac{11}{10} = \frac{100 + R}{100}$

$\Rightarrow 1100 = 10\,(100 + R)$

$\Rightarrow 1100 = 1000 + 10R$

$\Rightarrow 100 = 10R \quad \Rightarrow R = 10$

Hence, the required rate of interest is 10% per annum.

4. Given : CI = ₹ 331, P = ₹ 1000 and R = 10% per annum.

$\therefore CI = P\left[\left(1+\frac{R}{100}\right)^n - 1\right]$

$$\Rightarrow ₹\ 331 = ₹\ 1000\left[\left(1+\frac{10}{100}\right)^n - 1\right]$$

$$\Rightarrow ₹\ \frac{331}{1000} = \left(\frac{11}{10}\right)^n - 1$$

$$\Rightarrow \frac{331}{1000} + 1 = \left(\frac{11}{10}\right)^n$$

$$\Rightarrow \frac{1331}{1000} = \left(\frac{11}{10}\right)^n$$

$$\Rightarrow \left(\frac{11}{10}\right)^3 = \left(\frac{11}{10}\right)^n$$

$\Rightarrow 3 = n$ [Bases are same]

Hence, the required time is 3 years.

5. Similar work to be done as Q.3. of this Exercise.

6. Let the required sum of money be ₹ P.

Then $SI = \frac{P \times R \times T}{100} = \frac{P \times 10 \times 3}{100} = \frac{30P}{100} = ₹\ \frac{3P}{10}$

$$\therefore\ CI = P\left[\left(1+\frac{R}{100}\right)^n - 1\right]$$

$$= P\left[\left(1+\frac{10}{100}\right)^3 - 1\right] = P\left[\left(\frac{11}{10}\right)^3 - 1\right] = P\left[\frac{1331}{1000} - 1\right] = P\left[\frac{1331 - 1000}{1000}\right] = ₹\ \frac{331P}{1000}$$

According to the question,

CI – SI = ₹ 31

$$\Rightarrow ₹\ \frac{331P}{1000} - ₹\ \frac{3P}{10} = ₹\ 31$$

$$\Rightarrow ₹\ \frac{331P - 300P}{1000} = ₹\ 31$$

$\Rightarrow$ 31 P = ₹ 31000

$\Rightarrow$ P = ₹ 31000 ÷ 31 = ₹ 1000.

Hence, the required sum of money is ₹ 1000.

7. Given : P = ₹ 10,752, A = ₹ 15,309 and n = 3 years.

$$\therefore\ A = P\left(1+\frac{R}{100}\right)^n = ₹\ 15{,}309 = 10{,}752\left(1+\frac{R}{100}\right)^3$$

$\Rightarrow \frac{15309}{10752} = \left(\frac{100 + R}{100}\right)^3 \qquad \Rightarrow \frac{729}{512} = \left(\frac{100 + R}{100}\right)^3$ [Dividing by 21]

$$\Rightarrow \left(\frac{9}{8}\right)^3 = \left(\frac{100 + R}{100}\right)^3$$

$\Rightarrow \frac{9}{8} = \frac{100 + R}{100}$ [Powers are same]

$\Rightarrow$ 900 = 800 + 8 R
$\Rightarrow$ 900 – 800 = 8 R
$\Rightarrow$ 100 = 8 R
$\Rightarrow$ R = 100 ÷ 8 = 12.5.
Hence, the required rate of interest is 12.5%.

8. Given : P = ₹ 5000, A = ₹ 5955.08 and R = 12% per annum.
As the interested semi annually, so R = 12% ÷ 2 = 6% and n = 2n.

$\therefore A = P\left(1+\frac{R}{100}\right)^n$

$\Rightarrow ₹\ 5955.08 = ₹\ 5000\left(1+\frac{6}{100}\right)^{2n}$

$\Rightarrow ₹\ \frac{5955.08}{5000} = \left(\frac{106}{100}\right)^{2n}$

$\Rightarrow \frac{595508}{500000} = \left(\frac{106}{100}\right)^{2n}$

$\Rightarrow \frac{148877}{125000} = \left(\frac{106}{100}\right)^{2n}$ [Dividing the terms of $\frac{595508}{500000}$ by 4]

$\Rightarrow \left(\frac{53}{50}\right)^3 = \left(\frac{106}{100}\right)^{2n}$

$\Rightarrow \left(\frac{53}{50}\right)^3 = \left(\frac{53}{50}\right)^{2n}$

$\Rightarrow 3 = 2n \quad \Rightarrow n = 1\frac{1}{2}$ [Bases are same.]

Hence, the required time is $1\frac{1}{2}$ years.

9. Given : A = ₹ 8103.375, R = 5% per annum and n = 3 years
Let the sum of money borrowed be ₹ P.

Then $A = P\left(1+\frac{R}{100}\right)^n$

$\Rightarrow ₹\ 8103.375 = P\left(1+\frac{5}{100}\right)^3$

$\Rightarrow ₹\ 8103.375 = P\left(\frac{21}{20}\right)^3$

$\Rightarrow ₹\ 8103.375 = \frac{9261\ P}{8000}$

$\Rightarrow ₹\ \frac{8103.375 \times 8000}{9261} = P$

$\therefore$ P = ₹ 7000
Hence, the sum of money borrowed by Mukesh is ₹ 7000.

10. Given : A = ₹ 4759.04, n = 1 year and R = 8% compounded half yearly.

As the interest is compounded half-yearly, so n = 2 half years and R = 4% per half year.

Let the required sum be ₹ P.

Then $A = P\left(1+\frac{R}{100}\right)^n$

$\Rightarrow$ ₹ $4759.04 = P\left(1+\frac{4}{100}\right)^2$

$\Rightarrow$ ₹ $4759.04 = P\left(\frac{26}{25}\right)^2$

$\Rightarrow$ ₹ $4759.04 = P\times\frac{676}{625}$

$\Rightarrow$ P = ₹ $\frac{4759.04\times 625}{676}$ = ₹ 4400.

Hence, the required sum of money is ₹ 4400.

Revision Exercise

1. (a) Given : P = ₹ 6000, R = 5% per annum and n = 3 years

$\therefore A = P\left(1+\frac{R}{100}\right)^n$ = ₹ $6,000\left(1+\frac{5}{100}\right)^3$ = ₹ $6,000\left(\frac{21}{20}\right)^3$

= ₹ $6,000\times\frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}$ = ₹ 6945.75.

$\therefore$ Compound interest = A – P = ₹ 6945.75 – 6000 = ₹ 945.75.

Hence, the amount and compound interest are Rs. 6945.75 and Rs. 945.75 respectively.

(b) Similar work to be done.

(c) Similar work to be done.

2. Given : P = ₹ 74000, R = 2.5% per annum and n = 2 years.

$\therefore A = P\left(1+\frac{R}{100}\right)^n$ = ₹ $74,000\left(1+\frac{25}{100}\right)^2$ = ₹ $74,000\left(\frac{41}{40}\right)^2$ = ₹ $\frac{74,000\times 41\times 41}{40\times 40}$ = ₹ 77746.25

Hence, the amount that Sumit has to return is ₹ 77746.25.

3. Given : P = ₹ 4000, R = 10% per annum and n = $1\frac{1}{2}$ years.

As the interest is compounded half yearly, so R = 5% per half year and n = $\frac{3}{\cancel{2}}\times\cancel{2}$ = 3 half years.

$\therefore A = P\left(1+\frac{R}{100}\right)^n$ = ₹ $4,000\left(1+\frac{5}{100}\right)^3$ = ₹ $4,000\left(\frac{21}{20}\right)^3$ = ₹ $4,000\times\frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}$ = ₹ 4630.50.

$\therefore$ Compound interest = A – P = ₹ 4630.50 – ₹ 4000 = ₹ 630.50.

Hence, the required compound interest is ₹ 630.50 and the total amount to be paid is Rs. 4630.50 and the total amount to be paid is ₹ 4630.50.

4. Amount for 2 years :

A = ₹ $46,400\left(1+\frac{15}{100}\right)^3$ = ₹ $46,400\left(\frac{23}{20}\right)^3$ = ₹ $46,400\times\frac{23}{20}\times\frac{23}{20}$ = ₹ 61364.

Now interest for 4 months = ₹ $\dfrac{61364 \times 15 \times 14}{12 \times 100}$ = ₹ 3068.20.

Total amount = ₹ 61364 + ₹ 3068.20 = ₹ 64432.20.

Hence, Kunal will pay ₹ 64432.20 after the given period.

5. Here, P = 20000, R_1 = 10%, R_2 = 10% and R_3 = 10%

$\therefore$ Count of bacteria at the end of 3 hours,

$A = 20,000\left(1+\dfrac{10}{100}\right)\left(1-\dfrac{10}{100}\right)\left(1+\dfrac{10}{100}\right)$ = ₹ $20,000 \times \dfrac{11}{10} \times \dfrac{9}{10} \times \dfrac{11}{10}$ = 21,780.

Hence, the count of bacteria after 3 hours is 21,780.

6. Amount Sumeeta got, A = ₹ $6,400\left(1+\dfrac{10}{100}\right)^3$ = ₹ $6,400\left(\dfrac{11}{10}\right)^3$

= ₹ $6,400 \times \dfrac{11}{10} \times \dfrac{11}{10} \times \dfrac{11}{10}$ = ₹ 8518.40.

Interest of Geeta = $\dfrac{P \times R \times T}{100} = \dfrac{6400 \times 10 \times 3}{100}$ = ₹ 1920.

Amount Geeta got, A = ₹ 6400 + ₹ 1920 = ₹ 8320.

As ₹ 8518.40 > ₹ 8320, so Sumeeta got greater amount.

Difference between the two amounts = ₹ 8518.40 – ₹ 8320 = ₹ 198.40.

Hence, Sumeeta got ₹ 198.40 more than Geeta.

7. Here, P = ₹ 8000, A = ₹ 10648 and R = 10% per annum.

$\therefore A = P\left(1+\dfrac{R}{100}\right)^n$

$\Rightarrow$ ₹ $10648 = 8000\left(1+\dfrac{10}{100}\right)^2$

$\Rightarrow \dfrac{10648}{8000} = \left(\dfrac{11}{10}\right)^n$

$\Rightarrow \dfrac{1331}{1000} = \left(\dfrac{11}{10}\right)^n$ [Dividing by 8]

$\Rightarrow \left(\dfrac{11}{10}\right)^3 = \left(\dfrac{11}{10}\right)^n \quad \therefore n = 3$ [Bases are same.]

Hence, the required time is 3 years.

8. As the interest is compounded half yearly, so R = 4% per half year and n = 5 half years.

$\therefore A = P\left(1+\dfrac{R}{100}\right)^n$ = ₹ $5,000\left(1+\dfrac{4}{100}\right)^5$

= ₹ $5,000\left(\dfrac{26}{25}\right)^5$ = ₹ $5,000 \times \dfrac{26}{25} \times \dfrac{26}{25} \times \dfrac{26}{25} \times \dfrac{26}{25} \times \dfrac{26}{25}$ = ₹ 6083.26.

$\therefore$ CI = A – P = ₹ 6083.26 – ₹ 5000 = ₹ 1083.26.

$\therefore$ SI = $\dfrac{P \times R \times T}{100}$ = ₹ $\dfrac{5000 \times 4 \times 5}{100}$ = ₹ 1000.

9. $CI = P\left[\left(1+\frac{R}{100}\right)^n - 1\right] = ₹\ 36,000\left[\left(1+\frac{12}{100}\right)^2 - 1\right] = ₹\ 36,000\left[\left(\frac{28}{25}\right)^2 - 1\right] = ₹\ 36,000\left[\frac{784}{625} - 1\right]$

$= ₹\ 36,000 \times \frac{159}{625}$ = ₹ (57.60 × 15.9) = ₹ 9158.40.

Hence, the compound interest is ₹ 9158.40.

10. Similar work to be done as Q.8. above.

11. Here, P = 48000, A = 55566 and n = 3.

$\therefore A = P\left(1+\frac{R}{100}\right)^n$

$\Rightarrow 55566 = 48,000\left(1+\frac{R}{100}\right)^3$

$\Rightarrow \frac{55566}{48000} = \left(1+\frac{R}{100}\right)^3$

$\Rightarrow \frac{9261}{8000} = \left(1+\frac{R}{100}\right)^3$ [Dividing by 6]

$\Rightarrow \left(1+\frac{R}{100}\right)^3 = \left(\frac{21}{20}\right)^3$

$\Rightarrow 1+\frac{R}{100} = \frac{21}{20}$ [Powers are same.]

$\Rightarrow \frac{R}{100} = \frac{21}{20} - 1$

$\Rightarrow R = 100 \times \frac{1}{20} = 5\%$

Hence, the annual rate of growth of the factory is 5%.

12. Here, P = 500000, R = 6% per annum and n = 3 years.

$\therefore$ Population of the town after 3 years,

$A = P\left(1+\frac{R}{100}\right)^3 = 5,00,000\left(1+\frac{6}{100}\right)^3 = 5,00,000\left(\frac{53}{50}\right)^3 = 5,00,000 \times \frac{53}{50} \times \frac{53}{50} \times \frac{53}{50} = 595508.$

Hence, the population of the town after 3 years is 595508.

13. Here, P = ₹ 520000, R = 10% per annum and n = 2 years.

$\therefore$ The value of the car after 2 years,

$A = P\left(1-\frac{R}{100}\right)^n = ₹\ 5,20,000\left(1-\frac{10}{100}\right)^2 = ₹\ 5,20,000\left(\frac{9}{10}\right)^2 = ₹\ 5,20,000 \times \frac{9}{10} \times \frac{9}{10}$

= ₹ 5200 × 81 = ₹ 4,21,200.

Hence, the value of the car after 2 years will be ₹ 4,21,200.

Multiple Choice Questions

1. Simple interest in 2 years = ₹ 400 [Given]

∴ Simple interest for 1 year = ₹ 200

∴ Compound interest for 1 year = ₹ 200

Given compound interest for 2 years = ₹ 410.

∴ Compound interest for second year = ₹ 410 – ₹ 200 = ₹ 210.

∴ Interest on ₹ 200 for 1 year = ₹ 210 – ₹ 200 = ₹ 10.

Now, we know that :

$$\text{Interest} = \frac{P \times R \times T}{100}$$

$$\Rightarrow 10 = \frac{200 \times R \times 1}{100}$$

$$\Rightarrow 2R = 10$$

$$R = 10 \div 2 = 5\%$$

Hence, the correct option is (c).

2. Let the sum of money be ₹ P.

$$\text{Then CI} = P\left[\left(1+\frac{R}{100}\right)^n - 1\right]$$

$$\Rightarrow ₹\ 510 = P\left[\left(1+\frac{12.5}{100}\right)^2 - 1\right]$$

$$\Rightarrow ₹\ 510 = P\left[\left(1+\frac{1}{8}\right)^2 - 1\right]$$

$$\Rightarrow ₹\ 510 = P\left[\left(\frac{9}{8}\times\frac{9}{8}\right) - 1\right]$$

$$\Rightarrow ₹\ 510 = P\left[\frac{81 \quad 64}{64}\right] = \frac{17P}{64}$$

$$\Rightarrow ₹\ 510 = \frac{\cancel{510}^{30} \times 64}{\cancel{17}} = ₹\ 1920$$

$$\text{Now, SI} = \frac{P \times R \times T}{100} = ₹\ \frac{1920 \times 12.5 \times 2}{100} = ₹\ 480.\text{s}$$

Hence, the correct option is (c).

3. Compound interest for 1 year = Simple interest for 1 year = ₹ $\frac{10,0\cancel{00} \times 12 \times 1}{\cancel{100}}$ = ₹ 1200.

∴ Amount = ₹ 10000 + ₹ 1200 = ₹ 11200

$$\text{Now interest for half year} = \frac{P \times R \times T}{100} = ₹\ \frac{11,200 \times 12 \times 1}{100 \times 2} = ₹\ 672.$$

∴ Compound interest for 15 years = ₹ 1200 + ₹ 672 = ₹ 1872.

Hence, the correct option is (c).

4. $A = ₹\ 5000\left(1 + \frac{8}{100}\right)^2 = ₹\ 5000\left(\frac{27}{25}\right)^2 = ₹\ \frac{5000 \times 27 \times 27}{25 \times 25} = ₹\ 5832.$

Hence, the correct option is (d).

5. $A = P\left(1+\frac{R}{100}\right)^n = ₹\ 24,000\left(1+\frac{5}{100}\right)^2 = ₹\ 24,000\left(\frac{21}{20}\right)^2 = ₹\ 24,000 \times \frac{21}{20} \times \frac{21}{20} = ₹\ 26460.$

Hence, the correct option is (b).

6. $A = P\left(1+\frac{R}{100}\right)$

$\Rightarrow 2662 = 2000\left(1+\frac{R}{100}\right)^3$

$\Rightarrow \frac{2662}{2000} = \left(1+\frac{R}{100}\right)$

$\Rightarrow \frac{1331}{1000} = \left(1+\frac{R}{100}\right)^3 \Rightarrow \left(\frac{11}{10}\right)^3 = \left(1+\frac{R}{100}\right)^3$

$\Rightarrow \frac{11}{10} = 1+\frac{R}{100}$ [Powers are same.]

$\Rightarrow \frac{11}{10} - 1 = \frac{R}{100}$

$\Rightarrow \frac{1}{10} = \frac{R}{100} = \frac{\cancel{100}^{10}}{\cancel{10}} = R$

$\therefore R = 10\%$

Hence, the correct option is (a).

7. $CI = P\left[\left(1+\frac{R}{100}\right)^n - 1\right]$

$\Rightarrow P\left[\left(1+\frac{5}{100}\right)^2 - 1\right] = ₹\ 82$

$\Rightarrow P\left[\left(\frac{21}{20}\right)^2 - 1\right] = ₹\ 82$

$\Rightarrow P\left[\frac{441-400}{400}\right] = ₹\ 82$

$\Rightarrow 41P = ₹\ 82 \times 400$

$\Rightarrow P = ₹\ \frac{\cancel{82}^2 \times 400}{\cancel{41}_1} = ₹\ 800.$

$\therefore SI = \frac{P \times R \times T}{100} = ₹\ \frac{\cancel{800}^8 \times 5 \times 2}{\cancel{100}} = ₹\ 80.$

Hence, the correct option is (c).

8. As the interest is compounded quarterly.

$\therefore R = 8\% \div 4 = 2\%$ and $n = 2$ years $= 2 \times 4 = 8$ quarter years.

$\therefore CI = ₹\ 8000\left[\left(1+\frac{2}{100}\right)^8 - 1\right]$

$$CI = ₹\,800\left[\left(\frac{51}{50}\right)^{8} - 1\right]$$

Hence, the correct option is (b).

9. See Answers given in the book.

10. $A = P\left(1+\frac{R}{100}\right)^{n}$

$\Rightarrow 2178 = 1800\left(1+\frac{R}{100}\right)^{2}$

$\Rightarrow \frac{2178}{1800} = \left(1+\frac{R}{100}\right)^{2}$

$\Rightarrow \frac{1089}{900} = \left(1+\frac{R}{100}\right)^{2}$ [Dividing by 2]

$\Rightarrow \left(\frac{33}{30}\right)^{2} = \left(1+\frac{R}{100}\right)^{2}$

$\Rightarrow \frac{33}{30} = 1+\frac{R}{100}$ [Power are same.]

$\Rightarrow \frac{R}{100} = \frac{33-30}{30} = \frac{3}{30}$

$\Rightarrow 30R = 300$

$\Rightarrow R = 300 \div 30 = 10\%$

Hence, the correct option is (c).

Mental Maths

A. See Answers given in the book.

B. See Answers given in the book.

Higher Order Thinking Skills (HOTS)

1. According to the question.

$P\left(1+\frac{10}{100}\right)^{n} > 2^{8}$

$\therefore \left(\frac{11}{10}\right)^{n} > 2$

Now, $\left(\frac{11}{10}\times\frac{11}{10}\times\frac{11}{10}\times\frac{11}{10}\times\frac{11}{10}\times\frac{11}{10}\times\frac{11}{10}\times\frac{11}{10}\right) > 2$

$\therefore n = 8$

Hence, the least number of complete years is 8.

2. When the time is 1 year, both the compound interest and the simple interest are equal.

3. Present population of the town,

$$A = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right) = 6000\left(1+\frac{5}{100}\right)\left(1+\frac{4}{100}\right)\left(1+\frac{2}{100}\right)$$

$= 6000 \times \frac{21}{20} \times \frac{26}{25} \times \frac{51}{50} = 66830.4 = 66830.$

Increase in population = 66830 – 60000 = 6830.

Increase percentage $= \frac{6830 \times 100}{60000}\% = 11.384\%$

Hence, the increase percentage in population of the town is 11.384%.

12 Direct and Inverse Variations

Exercise 11.1

1. (a) $\frac{x}{y} = \frac{2}{5} = \frac{4}{10} = \frac{18}{45} = \frac{14}{35} = \frac{16}{40} = \frac{2}{5}$ (constant)

   Hence, x and y vary directly.

   (b) Here, $\frac{3}{6} = \frac{5}{10} = \frac{7}{14} = \frac{1}{2}$ (Constant)

   But $\frac{6}{18} = \frac{1}{3}$ and $\frac{2}{10} = \frac{1}{5}$

   Hence, x and y do not vary directly.

2. (a) As x and y vary directly,

   $\therefore \frac{x}{y} = k$ (constant)

   $k = \frac{4}{28} = \frac{1}{7}.$

   Now $\frac{9}{y_1} = \frac{1}{7} \Rightarrow y_1 = 7 \times 9 = 63$

   $\frac{x_1}{21} = \frac{1}{7} \Rightarrow x_1 = \frac{21}{7} = 3$

   $\frac{15}{y_2} = \frac{1}{7} \Rightarrow y_2 = 15 \times 7 = 105$

   Hence, $x_1 = 3$, $y_1 = 63$ and $y_2 = 105$.

   (b As x and y vary directly.

   $\therefore \frac{x}{y} = k$ (constant)

   $\therefore k = \frac{5}{6}$

   $\frac{x_1}{54} = \frac{5}{6} \Rightarrow x_1 = \frac{\cancel{54}^{9} \times 5}{\cancel{6}_1} = 45.$

   $\frac{x_2}{12} = \frac{5}{6} \Rightarrow x_2 = \frac{\cancel{12}^{2} \times 5}{\cancel{6}_1} = 10.$

$\frac{50}{y_1} = \frac{5}{6} \Rightarrow = \frac{\cancel{50}^{10} \times 6}{\cancel{5}_1} = 60.$

Hence, $x_1 = 45$, $x_2 = 10$ and $y_1 = 60$.

3. Let the cost of 14 books be ₹ x. Then we get :

Number of books	14	8
Cost (in ₹)	x	540

$\therefore \frac{14}{x} = \frac{8}{540}$

$\Rightarrow 8x = 540 \times 14$

$\Rightarrow x = \frac{540 \times 14}{8} = 945.$

Hence, the cost of 14 books is ₹ 945.

4. Let Pushpendra will earn ₹ x. Then, we get :

Number of hours	3	42
Earning (in ₹)	1050	x

$\therefore \frac{3}{1050} = \frac{42}{x}$

$x = \frac{1050 \times \cancel{42}^{14}}{\cancel{3}_1} = 1050 \times 14 = 14{,}700.$

Hence, Pushpendra will earn ₹ 14,700 in 42 hours.

5. Let the length of the shorter piece of wire be x. Then, we get :

Length of shorter piece (in cm)	2	x
Length of longer piece (in cm)	3	81

$\therefore \frac{x}{81} = \frac{2}{3}$

$\Rightarrow x = \frac{\cancel{81}^{27} \times 2}{\cancel{3}_1} = 27 \times 2 = 54$ cm.

Hence, the length of the shorter piece of wire will be 54 cm.

6. Let length of the shadow cost by another tree be x and the height of the tree which costs shadow of 6 m by y.

Height of tree (in cm)	510	1080	y	
Length of shadow (in cm)	3060	x	600	1111

Clearly, it is a case of direct variation.

(a) $\frac{510}{3060} = \frac{1080}{x}$

$\Rightarrow 510\,x = 1080 \times 3060$

$\Rightarrow x = \frac{1080 \times \cancel{3060}^{6}}{\cancel{510}_1} = 1080 \times 6$ cm $= 6480$ cm $= 6$ m 480 cm. $= 64$m 80 cm.

Hence, the length of the shadow is 6m 480 cm.

(b) $\frac{510}{3060} = \frac{y}{600}$

$\Rightarrow y \times 3060 = 600 \times 510$

$\Rightarrow y = \frac{600 \times \cancel{510}^{1}}{\cancel{3060}_{6}} = \frac{600}{6} = 100 \text{ cm} = 1 \text{ m}.$

Hence, the height of the tree is 1 m.

7. Let the distance travelled by the car be d and the time taken to travel 30 km be t. Then we get.

Distance travelled (in km)	90	d	30
Time taken (in min)	60	40	t

Clearly, it is the case of direct variation.

(a) $\frac{90}{60} = \frac{d}{40}$

$d = \frac{\cancel{90}^{3} \times 40}{\cancel{60}_{2}} = \frac{3 \times \cancel{40}^{20}}{\cancel{2}_{1}} = 3 \times 20 = 60 \text{ km}.$

Hence, the distance travelled by the car in 40 minutes is 60 km.

(b) $\frac{90}{60} = \frac{30}{t}$

$\Rightarrow t = \frac{60 \times \cancel{30}^{1}}{\cancel{90}_{3}} = \frac{\cancel{60}^{20}}{\cancel{3}_{1}} = 20 \text{ minutes}.$

Hence, the time taken by the car to travel 30 km in 20 minutes.

8. Let the earning of the worker be x and the number of days be y. Then we get :

Earning (in ₹)	400.25	x	11207
Number of days	5	30	y

Clearly, it is a case of direct variation.

(a) $\frac{400.25}{5} = \frac{x}{30}$

$\Rightarrow 5x = 400.25 \times 30$

$\Rightarrow x = \frac{400.25 \times \cancel{30}^{6}}{\cancel{5}_{1}} = 400.25 \times 6 = ₹\ 2401.50.$

Hence, the worker will get ₹ 2401.50 in the month of April.

(b) $\frac{400.25}{5} = \frac{11207}{y}$

$\Rightarrow 400.25\, y = 11207 \times 5$

$\Rightarrow y = \frac{11207 \times 5}{400.25} = 140 \text{ days}.$

9. (a) We have :

x	¾	12
y	10	y

$$\therefore \frac{\frac{3}{4}}{10} = \frac{12}{y}$$

$$\Rightarrow \frac{3}{40} = \frac{12}{y}$$

$$\Rightarrow 3y = 40 \times 12$$

$$\Rightarrow y = \frac{40 \times \cancel{12}^{4}}{\cancel{3}_{1}} = 40 \times 4 = 160$$

(b) We have :

v	4	x
t	9	21

$$\therefore \frac{4}{9} = \frac{x}{21} \qquad \Rightarrow 9x = 84$$

$$x = \frac{\cancel{84}^{28}}{\cancel{9}_{3}} = \frac{28}{3} = 9\frac{1}{3}.$$

10. Let the required distance be x cm. The, we get :

Distance on map (in cm)	1	7
Actual distance (in cm)	30000000	x

Clearly, it is a case of direct variation.

$$\therefore \frac{1}{30000000} = \frac{7}{x}$$

$$\Rightarrow x = 7 \times 30000000 = 210000000 \text{ cm}$$

$$= \frac{21\cancel{0000000}}{\cancel{100} \times \cancel{1000}} \text{ km} = 2100 \text{ km}$$ [1 m = 100 cm and 1 km = 1000 m]

Hence, the actual distance between the two cities is 2100 km.

11. Let the number of bottles that would be served be x. Then we get :

Number of bottles	8	x
Number of persons	6	42

Clearly, it is a case of direct variation.

$$\therefore \frac{8}{6} = \frac{x}{42}$$

$$\Rightarrow 6x = 42 \times 8$$

$$\Rightarrow x = \frac{\cancel{42}^{7} \times 8}{\cancel{6}_{1}} = 7 \times 8 = 56 \text{ bottles}$$

Hence, the required number of bottles is 56 bottles.

Exercise 11.2

1. (a) We know that if x and y vary inversely, then x y = k (constant)
Now, 3 × 9 = 27, 6 × 4 = 24, 12 × 2 = 24 and 3 × 8 = 24.
As the product in all cases is not constant, so x and y do not vary inversely.

(b) Similar work to be done.

2. As a and b vary inversely,

$\therefore\ 5 \times b_1 = 4 \times 20 \quad \Rightarrow b_1 = \frac{80}{5} = 16$

$a_1 \times 10 = 4 \times 20 \quad \Rightarrow a_1 = \frac{4 \times \cancel{20}^{2}}{\cancel{10}_{1}} = 8.$

$40 \times b_2 = 4 \times 20 \quad \Rightarrow b_2 = \frac{4 \times 20}{40} = 2.$

$a_2 \times 80 = 4 \times 20 \quad \Rightarrow a_2 = \frac{4 \times 20}{80} = 1$

Hence, $a_1 = 8$, $a_2 = 1$, $b_1 = 16$ and $b_2 = 2$.

3. We have :

x	10	5
y	8	y1

As x and y vary inversely,

$\therefore\ 10 \times 8 = 5 \times y1$

$\Rightarrow\ y_1 = \frac{10 \times 8}{5} = 16.$

Hence, the value of y when x = 5 is 16.

4. Similar work to be done as Q.3 above.

5. Let the required number of days be x.

Then $15 \times x = 15 \times 9$

$\Rightarrow\ x = \frac{\cancel{15} \times 9}{\cancel{15}} = 9$

Hence, 45 workers can construct the wall in 9 days.

6. Let the required time be x hours. Then we get :

Speed (in km/h)	60	90
Time (in h)	3	x

Clearly, it is a case of inverse variation.

$\therefore\ 90 \times x = 60 \times 3$

$\Rightarrow\ x = \frac{60 \times 3}{90} = \frac{180}{90} = 2$ hours.

Hence, the required time taken by car is 2 hours.

7. Let the time taken by the athlete be x minutes. Then we get :

Speed (in km/ h)	35	32
Time (in h)	16	x

Clearly, it is a case of inverse variation.

$\therefore\ 32 \times x = 35 \times 16 \quad \Rightarrow\ x = \frac{35 \times \cancel{16}^{1}}{\cancel{32}_{2}} = \frac{35}{2} = 17\frac{1}{2}$ minutes.

Hence, the athlete will complete the race in $17\frac{1}{2}$ minutes .

8. (a) Remaining number of soldiers = 1200 – 300 = 900.

Let the food will last for x days. Then, we get

Number of soldiers (x)	1200	900
Number of days (y)	30	x

Clearly, it is a case of inverse variation.

$\therefore$ 900 × x = 1200 × 30

$\Rightarrow x = \frac{1200 \times 30}{900} = \frac{12^4 \times 30^{10}}{9_3} = 40$ days.

Hence, the food will last for 40 days.

(b) Here, the total number of soldiers = 1200 + 300 = 1500.

Let the food will last for x days. Then, we get :

Number of soldiers (x)	1200	1500
Number of days (y)	~~1500~~ 30	x

Clearly, it is a case of inverse variation.

$\therefore$ 1500 × x = 1200 × 30

$\Rightarrow x = \frac{1200 \times 30^2}{_1 1500} = 12 \times 2 = 24$ days.

Hence, the food will last for 24 days.

9. Let the speed that Nidhi should increase be x km/h.

As Nidhi wants to reach her school, five minutes before, So the time will be 20 min – 5 min = 15 min.

Speed (in km/h)	15	x
Time (in min)	20	15

Clearly, it is a case of inverse variation.

$\therefore$ x × 15 = 15 × 20

$\Rightarrow x = \frac{15 \times 20}{15} = 20$ km/h

$\therefore$ Speed increased = 20 km/h – 15 km/h = 5 km/h.

Hence, Nidhi should increase her speed 5 km/h to reach school 5 minutes before.

10. Total time of 8 periods = 40 min. × 8 = 320 minutes.

Recess time = 20 minutes

Remaining time = 320 minutes – 20 minutes = 300 minutes.

Number of periods = 6

$\therefore$ Duration of each period = $\frac{300^{50} \text{ minutes}}{6}$ = 50 minutes.

Hence, each period will be of duration of 50 minutes.

11. Amount of money of 64 trousers = ₹ 825 × 64 = ₹ 52,800.

New cost of 1 trouser = ₹ 825 + ₹ 231 = ₹ 1056.

Let the number of trouser to be bought be x. Then we get :

Number of trousers	Cost of 1 trouser (in ₹)	Total amount of money (in ₹)
64	825	52800
x	1056	52800

Clearly, it is a case of inverse variation.

$$\therefore\ 64\ x = \frac{52800 \times 52800}{825 \times 1056}$$

$$\Rightarrow\ x = \frac{52800 \times 52800}{825 \times 1056 \times 64} = 50.$$

Hence, 50 trousers can be bought.

12. Let the new speed be x km/h. Then we get :

Speed (in km/h)	48	x
Time (in h)	10	8

Clearly, it is a case of inverse variation.

$\therefore\ 8 \times x = 48 \times 10$

$$\Rightarrow\ x = \frac{\cancel{48}^{6} \times 10}{\cancel{8}_{1}} = 60$$

$\therefore$ New speed = 60 km/h

Increase in speed = 60 km/h – 48 km/h = 12 km/h

Hence, the train should increase its speed by 12 km/h.

Revision Exercise

1. Similar work to be done as Q. 2. of Exercise 11.1.
2. Similar work to be done as Q. 2. of Exercise 11.2.
3. Similar work to be done as Q. 1. of Exercise 11.1. and Q.1. of Exercise 11.2.
4. (a) Side of a square and its area vary directly.
   (b) Speed of a train and time taken to cover a fixed distance vary inversely.
   (c) As the number of pens increases, their cost also increases. Thus, it is a case of direct variation.
   (d) With the uniform speed, the distance and time vary directly. Thus, it is a case of direct variation.
5. Let the required number of days be x. Then we get :

Number of workers	8	6
Number of days	24	x

Clearly, it is a case of inverse variation.

$\therefore\ 6x = 24 \times 8$

$$\Rightarrow\ x = \frac{\cancel{24}^{4} \times 8}{\cancel{6}_{1}} = 32 \text{ days}$$

6. Similar work to be done as Q. 5. above.

7. Clearly, the remaining food is sufficient for 150 soldiers for 45 – 10 = 35 days.

Remaining soldiers	150	125
Number of days	45	x

It is 6 case of inverse variation.

$\therefore$ 125 × x = 150 × 45

$\Rightarrow x = \frac{150 \times 45}{125} = 54$ days.

Hence, the food will last in 54 days.

8. Let the required time be x minutes. Then we get :

Average speed	12	16
Time (in minutes)	20	x

Clearly, it is a case of inverse variation.

$\therefore$ 16 × x = 12 × 20

$\Rightarrow x = \frac{12 \times 20}{16} = \frac{240}{16} = 15$ minutes

Hence, Rajan will take 15 minutes to reach school.

9. Let Reena should read x hours a day. Then we get :

Reading time (in h)	4	x
Number of days	15	10

Clearly, it is a case of inverse variation.

$\therefore$ 10 × x = 15 × 4

$\Rightarrow x = \frac{15 \times 4}{10} = \frac{60}{10} = 6$ hours.

Hence, Reena should read 6 hours to complete the book.

10. (a) Let the distance between two cities on the map be x cm. Then we get :

Distance on map (in cm)	1	x
Actual distance (in cm)	5×10^6	25×10^6

Clearly, it is a case of direct variation. [250 km = 250 × 100 × 1000 cm]

$$\therefore \frac{x}{25 \times 10^6} = \frac{1}{5 \times 10^6} \Rightarrow x = \frac{25 \times \cancel{10}^6}{5 \times \cancel{10}^6} = \frac{25}{5} = 5 \text{ cm.}$$

Hence, the distance between two cities on the map in 5 cm.

(b) Given that the distance between two cities on the map = 3 cm.

Let the actual distance between two cities be x cm.

Then we get :

Distance on map (in cm)	1	3
Actual distance (in cm)	5×10^6	x

Clearly, it is a case of direct variation

$$\therefore \frac{1}{5 \times 10^6} = \frac{3}{x}$$

$$\Rightarrow x = 3 \times 5 \times 10^6 = 15 \times 10^6 \text{ cm} = \frac{15 \times 10\cancel{00000}}{\cancel{100} \times \cancel{1000}} = 15 \times 10 = 150 \text{ km.}$$

Hence, the actual distance between two cities is 150 km.

Multiple Choice Questions

1. Cost of 6 kg oranges = ₹ 360

$$\therefore \text{Cost of 15 kg oranges} = ₹\ \frac{\cancel{360}^{60} \times 15}{\cancel{6}} = ₹\ 00.$$

Hence, the correct option is (b).

2. Here, x y = 8 × 9 = 72.
Now 24 × 3 = 72, 1.2 × 6 = 7.2, 36 × 2 = 72 and 18 × 4 = 72.
Hence, the correct option is (b).

3. $\text{Speed} = \frac{\text{Distance}}{\text{Time}}$

$$\therefore \text{Distance} = \text{Speed} \times \text{Time} = \frac{\cancel{36} \times 10\cancel{00}}{\cancel{60} \times \cancel{60}} = 10 \times 12 = 120 \text{ m.}$$

Hence, the correct option is (d).

4. Height of the tree = $\frac{18 \times 11}{10} = \frac{198}{10}$ = 19.8 m. Hence, the correct option is (c).

5. If $\frac{a}{b}$ is a constant, then, a varies directly with b. Hence, the correct option is (b).

6. 25% of the work is done in 1 day.

$$\therefore 100\% \text{ work will be done in } \frac{1 \times \cancel{100}^{4}}{\cancel{25}} \text{ days} = 4 \text{ days.}$$

Hence, the correct option is (d).

7. Two quantities x and y are inversely proportional, if xy is constant.
Hence, the correct option is (b).

8. 8 men build a wall in 6 days.
∴ 1 man will build the wall in (6 × 8) days.

$$\therefore 6 \text{ men will build the wall in } \frac{\cancel{6} \times 8}{\cancel{6}} = 8 \text{ days.}$$

Hence, the correct option is (b).

9. Distance travelled by the car in 5 hours = 300 km

$$\text{Distance travelled by the car is 8 hours} = \frac{\cancel{300}^{60} \times 20}{\cancel{5}_1} = 60 \times 8 = 480 \text{ km}$$

Hence, the correct option is (b).

10. The actual distance between two places.

$$= 5 \times 20000000 = 100000000 \text{ cm} = \frac{100\cancel{000000}}{\cancel{100} \times \cancel{1000}} = 1000 \text{ km.}$$

Hence, the correct option is (d).

Mental Maths

A. See **Answers** given in the book.

B. See **Answers** given in the book.

Higher Order Thinking Skills (HOTS)

2. As a varies inversely as b^2, then $ab^2 = 0.3 \times 2 = 0.6$

$\therefore a = \frac{0.6}{b^2}$.

3. As p varies inversely as q, then pq = 2 xq

$\therefore x = \frac{p\not{q}}{2\not{q}} = \frac{p}{2}$. Hence, p will become half.

12 Quadrilaterals

Exercise 12.1

1. (i) We know that sum of the angles of a quadrilateral is 360°.

$\therefore x + x + 130° + 90° = 360°$

$\Rightarrow 2x + 220° = 360°$

$\Rightarrow 2x = 360° - 220° = 140°$ $\quad \Rightarrow x = \frac{140\Upsilon}{2} = 70°$

Hence, the value of x is 70°.

(ii) Sum of the angles of the quadrilateral = 360°

$\Rightarrow 2x + 110° + 60° = 360°$

$\Rightarrow 2x + 270° = 360°$

$\Rightarrow 2x = 360° - 270° = 90°$ $\quad \Rightarrow x = \frac{90\Upsilon}{2} = 45°$

Hence, the value of x is 45°.

2. (a) We have 80°, 120°, 70° and 90°

Their sum = 80° + 120° + 70° + 90° = 360°

Hence, these angles form a quadrilateral.

(b) We have 75°, 98°, 98° and 59°

Their sum = 75° + 98° + 98° + 59° = 330°

$\therefore$ Sum is not equal to 360°.

Hence, these angles cannot form a quadrilateral.

(c) We have 120°, 185°, 40° and 15°.

Their sum = 120° + 185° + 40° + 15° = 360°

Hence, these angles form a quadrilateral.

3. We have the three angles of a quadrilateral 105°, 45° and 60°.

Let the fourth angle of the quadrilateral be x°.

Then by the angle sum property of a quadrilateral,

105° + 45° + 60° + x° = 360°

210° + x° = 360°

$x° = 360° – 210° = 150°$.

Hence, the fourth angle of the quadrilateral is 150°.

4. From the given figure, $\angle QOR = 50°$, $\angle PQO = 90°$ and $\angle PRO = 90°$.

Let the $\angle ROQ$ be x°.

Then by the angle sum property of a quadrilateral

$\angle QOR + \angle OQP + \angle ORP + \angle RPQ = 360°$.

$50° + 90° + 90° + x° = 360°$

$230° + x° = 360°$

$x° = 360° – 230° = 130°$.

Hence, the measure of $\angle RPQ$ is 130°.

5. Given : ABCD is a quadrilateral in which DC || AB.

$\therefore$ Sum of the opposite angle of ABCD is 180°.

$\therefore \angle A + \angle C = 180°$

$\angle A + 140° = 180°$

$\angle A = 180° – 140° = 40°$

Also, $\angle B + \angle D = 180°$

$\Rightarrow \angle B + 100° = 180°$

$\Rightarrow \angle B = 180° – 100° = 80°$

Hence, $\angle A = 40°$ and $\angle B = 80°$.

6. Let the fourth angle of the quadrilateral be x°.

The $x° + 54° + 70° + 86° = 360°$ [Angle seen property of a quadrilateral]

$x° + 210° = 360°$

$x° = 360° – 210° = 150°$

Hence, the fourth angle of the quadrilateral is of 150°.

7. Let the fourth angle of the quadrilateral be x.

Then $x + 45° + 45° + 135° = 360°$ [Angle sum property of a quadrilateral]

$x + 225° = 360°$

$x = 360° – 225° = 135°$.

Hence, the fourth angle of the quadrilateral is of 135°.

8. Let one of the four equal angles of the quadrilateral be x.

Then $x + x + x + x = 360°$.

$4x = 360°$

$x = \frac{360Υ}{4} = 90°$.

Hence, each angle of the quadrilateral is of 90°.

9. Let the angles of the quadrilateral be 2x, 3x, 5x and 8x.

Then $2x + 3x + 5x + 8x = 360°$ [Angle sum property of a quadrilateral]

$18x = 360°$

$x = \frac{360°}{18} = 20°$

$\therefore 2x = 2 \times 20° = 40°, 3x = 3 \times 20° = 60°, 56x = 5 \times 20° = 100°$ and

$8x = 8 \times 20° = 160°$.

Hence, the angles of the quadrilateral are 40°, 60°, 120° and 160°.

10. Given : Average of angles of the quadrilateral is 80°.

$\therefore$ Sum of the three angles of the quadrilateral = $80° \times 3 = 240°$

$2x + 3x + 7x = 240°$

$12x = 240°$

$$x = \frac{240ϒ}{12} = 20°$$

$\therefore$ $2x = 2 \times 20° = 40°$, $3x = 3 \times 20° = 60°$ and $7x = 7 \times 20° = 140°$.

$\therefore$ Fourth angle of the quadrilateral = $360° - 240° = 120°$.

Hence, angles of the quadrilateral are 40°, 60°, 140° and 120°.

Exercise 12.2

1. (a) $70° + 110° = 180°$ [Co-interior angles]

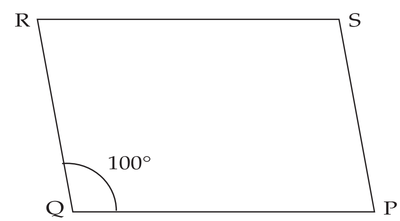
   Hence, this quadrilateral is a parallelogram.

   (b) As the opposite angle are equal, so it is a parallelogram.

   (c) As the opposite angles of the quadrilateral are not equal, so it is not a parallelogram.

   (d) As the opposite sides of the quadrilateral are equal, to it is a parallelogram.

2. Let PQRS be the quadrilateral.



Then $\angle Q = 100° = \angle S$ [Opposite angles]

Now $\angle P + \angle Q = 180°$ [Co-interior angles]

$\angle P + 100° = 180°$

$\angle P = 180° - 160° = 80°$.

Also, $\angle P = \angle R = 80°$ [Opposite angles]

Hence, the other three angles of the given quadrilateral are

$\angle P = \angle R = 80°$ and $\angle S = 100°$.

3. We know that the adjacent angles of a parallelogram are supplementary.

Let the adjacent angles of the quadrilateral be 4 x and 5 x.

Then $4x + 5x = 180°$

$$\Rightarrow 9x = 180° \qquad \Rightarrow x = \frac{180°}{9} = 20°$$

$\therefore$ $4x = 4 \times 20° = 80°$ and $5x = 5 \times 20° = 100°$.

Hence, angles of the quadrilateral are 80°, 100°, 80° and 100°.

4. From the given figure,

$\angle DAC = \angle ACB$ [Alternate angles]

$\Rightarrow 40° = y \quad \Rightarrow y = 40°$.

Now $\angle ABC + 70° = 180°$ [Linear pair]

$\Rightarrow \angle ABC = 180° - 70° = 110°$.

$\Rightarrow \angle ADC = \angle ABC$ [Opposite angles]

$\therefore$ $x = 110°$.

In ΔABC

$\angle A + \angle B + \angle C = 180°$ [Angle sum property of a triangle]

$z + 110° + 40° = 180°$

$z = 180° - 150° = 30°$

Hence, $x = 110°$, $y = 40°$ and $z = 30°$.

5. We know that adjacent angles of a parallelogram are supplementary.

$\therefore (8x + 115)° + (2x + 45)° = 180°$

$\Rightarrow 8x + 2x + 115° + 45° = 180°$

$\Rightarrow 10x = 180° - 160° = 20°$

$\Rightarrow x = 20° \div 10 = 2°$

$\therefore 8x + 115° = 8 \times 2° + 115° = 16° + 115° = 131°$

and $2x + 45° = 2 \times 2° + 45° = 4° + 45° = 49°$.

Also, angle opposite to 131° = 131° and that opposite to 49° = 49°.

Hence, the angles of the quadrilateral are 131°, 49°, 131° and 49°.

6. Let the shorter side of the parallelogram be x cm.

Then its longer side = (x + 7) cm.

$\therefore$ Perimeter of the parallelogram = 210 cm. [Given]

$\Rightarrow$ 2 (Sum of adjacent sides) = 210 cm

$\Rightarrow$ Sum of adjacent sides = 105 cm

$\Rightarrow x + x + 7 = 105$ cm

$\Rightarrow 2x = (105 - 7)$ cm = 98 cm

$\Rightarrow x = 98$ cm $\div 2 = 49$ cm.

Hence, the adjacent sides of the parallelogram are 49 cm and 49 cm + 7 cm = 56 cm.

7. Let the longer side of the parallelogram be x cm.

Then its shorter side = (x – 15) cm.

Next similar work to be done as Q. 6. above.

8. Let the smaller of the adjacent angles be x.

Then its longer side will be 3x.

$\therefore x + 3x = 180°$ [Adjacent angles are supplementary]

$4x = 180°$

$x = 180° \div 4 = 45°$

$\therefore 3x = 3 \times 45°\ 135°$

Hence, angles of the parallelogram are 45°, 135°, 45° and 135°.

9. From the given figure.

$\angle BAD = \angle BCD$ [Opposite angles]

$\therefore \angle BAD = 60°$

Now $\angle ABC + \angle BCD = 180°$ [Adjacent angles are supplementary]

$\therefore \angle ABC\ 180° - 60° = 120°$

Given that bisectors of A and B meet at 0.

$\therefore \angle BAO = \frac{1}{2} \angle BAD = \frac{1}{2} \times 60° = 30°$.

and $\angle ABO = \frac{1}{2} \angle ABC = \frac{1}{2} \times 120° = 60°$

In ΔAOB,

$\angle BAO + \angle ABO + \angle AOB = 180°$ [Angle sum property of a triangle]

$30° + 60° + \angle AOB = 180°$

$\angle ADB = 180° - 90° = 90°$

Hence, the measure of $\angle AOB$ is 90°.

10. Given : A parallelogram PQRS in which QPS = 75° and SQR = 65°.

$\therefore \angle SRQ = \angle SPQ$ [Opposite angles]

$\angle SRQ = 75°$. ...(i)

In Δ $\angle$SRO,

$\angle RSQ + \angle SRQ + \angle SQR = 180°$ [Angle sum property of a triangle]

$\Rightarrow \angle RSQ + 75° + 65° = 180°$

$\Rightarrow \angle RSQ + 140° = 180°$

$\Rightarrow \angle RSQ = 180° - 140° = 40°$

Hence, $\angle RSQ = 40°$ and $\angle SRO = 75°$.

Exercise 12.3

1. Let PQRS be the given rhombus with PR = 8 cm and SQ = 6 cm.

We know that diagonals of a rhombus bisect each other at right angles.

$\therefore$ PO = OR = 4 cm and QO = OS = 3 cm

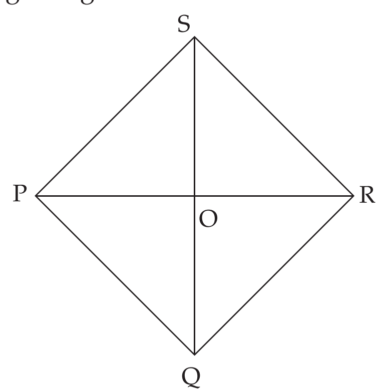
In right angles triangle POQ,

$PQ^2 = PO^2 + OQ^2 = (4 \text{ cm})^2 + (3 \text{ cm})^2$

$= 16 \text{ cm}^2 + 9 \text{ cm}^2 = 25 \text{ cm}^2$

$PQ = \sqrt{25 \text{ cm}^2} = 5$ cm.

Hence, the side of the rhombus is 5 cm.



2. From the figure,

$\angle DBC = \angle ADB$ [Alternate angles]

$\therefore \angle DBC = 60°$

As diagonals of a rhombus bisect each other at right angles.

$\therefore \angle BOC = 90°$

Now, in ΔBOC,

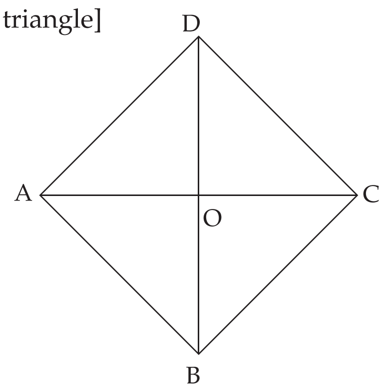
$\angle BOC + \angle OBC + \angle BCO = 180°$ [Angle sum property of a triangle]

$90° + 60° + \angle BCD = 180°$

$\angle BCD = 180° - 150° = 30°$

$\therefore \angle ACB = \angle BCD = 30°$

Hence, $\angle ACB$ is of 30°.



3. Let ABCD be the given rhombus with AC = AD.

As all sides of a rhombus are equal,

$\therefore$ AC = AD = CD.

$\therefore$ ΔACD is an equilateral triangle.

In ΔACD,

$\Rightarrow \angle ADC + \angle ACD + \angle CAD = 180°$ [Angle sum property of a triangle]

$\Rightarrow \angle ADC + \angle ADC + \angle ADC = 180°$ [Angles of an equilateral triangles are equal.]

$\Rightarrow 3\angle ADC = 180°$

$\Rightarrow \angle ADC = 180° \div 3 = 60°$

Now, $\angle ADC + \angle BCD = 180°$ [Adjacent angles are supplementary.]

$\Rightarrow 60° + \angle BCD = 180°$

$\Rightarrow \angle BCD = 180° - 60° = 120°$

$\therefore \angle BAD = \angle BCD = 120°$ [Opposite angles]

Also, $\angle ADC = \angle ABC = 60°$ [Opposite angles]

Hence, the angles of the given rhombus are 60°, 120°, 60° and 120°.

4. All angles of a square and a rectangle are equal.
Adjacent sides of a square are equal but of a rectangle are not equal.
Hence, the special quadrilateral is a rectangle.

5. We know that the diagonals of a rectangle are equal and bisect each other.

$\therefore OP = OR = 3x + 1$

$\therefore OP + OR = PR = 2(3x + 1)$

$PR = (6x + 2)$ cm ...(i)

Also $OS = OQ = 2x + 3$

$\therefore QS = OS + OQ = 2(2x + 3) = (4x + 6)$ cm ...(ii)

Now, PR = QS [Diagonals are equal]

$\Rightarrow 6x + 2 = 4x + 6$ [From (i) and (ii)]

$\Rightarrow 6x - 4x = 6 - 2 = 4$

$\Rightarrow 2x = 4$

$\Rightarrow x = 2$ cm

$\therefore PR = 6x + 2 = 6 \times 2 + 2 = 12 + 2 = 14$ cm.

Hence, PR is of 14 cm.

6. Let ABCD be the given rhombus with $\angle BAD = 60°$

Then $\angle BCD = \angle BAD = 60°$ [Opposite angles]

$\therefore \angle BAD + \angle ADC = 180°$ [Adjacent angles are supplementary]

$\Rightarrow 60° + \angle ADC = 180°$

$\Rightarrow \angle ADC = 180° - 60° = 120°$

As diagonals of the rhombus bisect the angles.

$\therefore \angle ADB + \angle BDC = 120°$

$\Rightarrow \angle ADB + \angle ADB = 120°$

$\Rightarrow 2\angle ADB = 120°$ $\Rightarrow \angle ADB = 120° \div 2 = 60°$

Similarly, $\angle ABD = 60°$

In ΔABD,

$\angle ABD + \angle ADB + \angle BAD = 180°$ [Angle sum property]

$60° + 60° + 60° = 180°$

$180° = 180°$

It means ΔACD is an equilateral triangle.

Similarly, ΔBCD is an equilateral triangle.

$\therefore \Delta ABD \approx \Delta BCD$

i.e., the shorter diagonal BD divides the rhombus ABCD into two equilateral triangles.

Hence, proved.

7. Let the diagonals AD and BD bisect each other at a point O. Then in $\varnothing$COD,

$x + 45° + 90° = 180°$ [Angle sum property of a triangle]

$x + 135° = 180°$

$x = 180° - 135° = 45°$

Hence, the value of x is 45°.

8. We know that every angle of a square is 90°. Also a diagonal of a square bisect the joining angles. From the figure, inΔPSR,

$\angle 1 = \angle 2 = 90° \div 2 = 45°$.

Hence, the value of $\angle SRP$ is 45°.

9. Given that AB || CD and base angles are equal each of 40°. So, it is an isosceles trapezium.

$\therefore \angle A + \angle D = 180°$ [Adjacent angles]

$\Rightarrow 40° + \angle D = 180°$

$\Rightarrow \angle D = 180° - 40° = 140°$

Similarly $\angle B + \angle C = 180°$

$\Rightarrow \angle C = 180° - 40° = 140°$.

Hence, angles of the trapezium are 40°, 140°, 40° and 140°.

10. Let PQRS be the given kite.

Then we know that diagonals of a kite intersect each other at right angles. Also, a kite has two pairs of adjacent equal sides.

$\therefore$ In kite PQRS,

PS = QS and SR = QR.

In $\varnothing$ORS

$\Rightarrow \angle ORS + \angle SOR + \angle OSR = 180°$ [Angle show property of a triangle]

$\Rightarrow 25° + 90° + \angle OSR = 180°$

$\Rightarrow \angle OSR = 180° - 115° = 65°$

$\therefore \angle OQR = \angle OSR = 65°$. [Angles opposite to equal sides]

(i) Now in $\varnothing$QRS

$\Rightarrow \angle QRS + \angle RSQ + \angle RQS = 180°$

$\Rightarrow \angle QRS + 65° + 65° = 180°$

$\Rightarrow \angle QRS = 180° - 130° = 50°$.

Hence, $\angle QRS$ is of 50°.

(ii) As PS = PQ

$\therefore \angle PSQ = \angle PQS = 40°$ [Angles formed by equal sides]

Now, in $\varnothing$QPS,

$\Rightarrow \angle QPS + \angle QSP + \angle PQS = 180°$.

$\Rightarrow \angle QPS + 40° + 40° = 180°$.

$\Rightarrow \angle QPS = 180° - 80° = 100°$.

Hence, $\angle QPS$ is of 100°.

(iii) $\angle RQP = \angle RQS + \angle PQS = 65° + 40° = 105°$

Revision Exercise

1. Let the given figure be the trapezium PQRS with PQ || RS and RT || PS.

(i) Given that PQ || RS, so the distance between PQ and RS is constant.

$\therefore$ RT = PS

Also PS || RT $\therefore$ PT = SR

As opposite sides of quadrilateral PSRT are equal and parallel, so PQRS is a parallelogram.

Hence, Proved.

(ii) As PQRS is parallelogram, so the opposite sides are equal, i.e., PS = RT

Hence, proved.

(iii) Given that PQ = QR and PQ || SR. Also RT || PS.

$\therefore$ PQ = RT = QR

$\therefore$ ΔQRT is an isosceles triangle.

Hence, proved.

2. Let the longer side of the rectangle be x an.

Then its shorter side = (x– 14) cm.

$\therefore$ Perimeter of the rectangle = 112 cm.

$\Rightarrow$ 2 (longer side + shorter side) = 112 cm.

$\Rightarrow$ $x + x - 14 = 112 \div 2 = 56$ cm

$\Rightarrow$ $2x = 56 + 14 = 70$ cm

$\Rightarrow$ $x = 70 \text{ cm} \div 2 = 35$ cm.

$\therefore$ $x - 14 = 35 \text{ cm} - 14 \text{ cm} = 21$ cm.

Hence, the sides of the rectangle are 35 cm and 21 cm.

3. We know that the diagonals of a square are equal and bisect to each other at right angles.

$\therefore$ In square ABCD:

OD = OC

$\Rightarrow$ $3x + 2 = 6x - 4$

$\Rightarrow$ $3x - 6x = -4 - 2$

$\Rightarrow -3x = -6$ $\Rightarrow x = 2$

$\therefore$ OD = $3x + 2 = 3 \times 2 + 2 = 8$ units

OC = $6x - 4 = 6 \times 2 - 4 = 12 - 4 = 8$ units

Now in ΔCOD,

$CD^2 = OD^2 + OC^2 = 8^2 + 8^2 = 64 + 64 = 128$ sq units

$CD = 2 \times 8^2 = 8\sqrt{2}$ units

Hence, sides of the given square are $8\sqrt{2}$ units each.

4. From the figure,

$y + 80° = 180°$ [Linear pair]

$y = 180° - 80° = 100°$

$x = y = 100°$ [Opposite angles of a parallelogram are equal]

$\angle QPR = \angle PRS = 20°$ [Alternate angles]

Now, in ΔPQR,

$z + \angle QPR + \angle PQR = 180°$ [Angle sum property of a triangle]

$\Rightarrow$ $z + 20° + 100° = 180°$

$\Rightarrow$ $z = 180° - 120° = 60°$

Hence, $x = y = 100°$ and $z = 60°$

5. We know that opposite sides of a rectangle are equal.

$\therefore$ 5y + 3 = 18

5y = 18 – 3 = 15

y = 15 ÷ 5 = 3 units

Also, 2x + 3 = 35

2x = 35 – 3 = 32

x = 32 ÷ 2 = 16 units

Hence, the values of x and y are respectively 16 units and 3 units.

6. See Answers given in the book.

7. Let the length and breadth of the parallelogram and respectively 3x and 2x.

Then perimeter of the parallelogram = 50 cm [Given]

$\Rightarrow$ 2 (length + breadth) = 50 cm

$\Rightarrow$ 3x + 2x = 50 ÷ 2 = 25 cm

$\Rightarrow$ 5x = 25 cm

$\Rightarrow$ x = 25 cm ÷ 5 = 5 cm

$\therefore$ 3x = 3 × 5 cm = 15 cm

and 2x = 2 × 5 cm = 10 cm

Hence, the dimensions of the parallelogram are 15 cm and 10 cm.

8. Let the angles of the quadrilaterals are x, 3x, 4x and 4x.

Then by the angle sum property of a quadrilateral,

x + 3x + 4x + 4x = 360°

$\Rightarrow$ 12x = 360°

$\Rightarrow$ x = 360° ÷ 12 = 30°

$\therefore$ 3x = 3 × 30° = 90° and 4x = 4 × 30° = 120°

Hence, the angles of the quadrilateral are 30°, 90°, 120° and 120°.

9. We know that diagonals of a rectangle are equal and bisect each other.

QO = QF

$\angle$FQO = 110° [Opposite angles]

Now in ΔFQO,

$\Rightarrow$ $\angle$QFO + $\angle$QOF + $\angle$FQO = 180° [Angle sum property]

$\Rightarrow$ $\angle$QFO + $\angle$QFO + 110° = 180° [Angle opposite to equal sides]

$\Rightarrow$ 2QFO = 180° – 110° = 70°

$\Rightarrow$ $\angle$QFO = 70° ÷ 2 = 35°

Hence, $\angle$QOF = $\angle$QFO = 35°.

10. Let ABCD be the given rhombus in which AC = 30 cm and BD = 16 cm intersecting at a point O.

We know that diagonals of a rhombus bisect each other at right angles.

$\therefore$ AO = OC = 30 cm ÷ 2 = 15 cm

BO = OD = 16 cm ÷ 2 = 8 cm

Now in ΔAOB,

$AB^2 = OA^2 + OB^2 = (15\text{ cm})^2 + (8\text{ cm})^2 = 225\text{ cm}^2 + 64\text{ cm}^2 = 289$ cm

$AB = \sqrt{289}$ cm = 17 cm.

Hence, the length of the side of the rhombus is 17 cm.

11. We know that opposite angles of a parallelogram are equal.

$\therefore$ In parallelogram PQRS, $\angle P = \angle R = 70°$

In parallelogram ABCD, $\angle B = \angle D = 100$

Now $\angle A + \angle B = 180°$ [Adjacent angles]

$\Rightarrow \angle A + 100° = 180°$

$\Rightarrow \angle A = 180° - 100° = 80°$.

In ΔAOR,

$\angle AOR + \angle OAR + \angle ORA = 180°$ [Angle sum property]

$\Rightarrow \angle AOR + 80° + 70° = 180°$

$\Rightarrow \angle AOR = 180° - 150° = 30°$

$\therefore \angle AOR + x = 180°$ [Linear pair]

$30° + x = 180°$

$x = 180° - 30° = 150°$

Hence, the measure of the angle x is 150°.

12. (i) From the given figure of parallelogram PQRS,

$\Rightarrow \angle Q = \angle S$ [Opposite angles]

$\Rightarrow \angle a = \angle Q = 50°$

$\therefore$ a = 50°

$\Rightarrow \angle SPR = \angle PRQ$ [Alternate angles]

c = 70°

$b = 180° - (60° + 90°) = 180° - 150° = 30°$

Hence, a = 50°, b = 30° and c = 70°.

(ii) From the figure of parallelogram PQRS,

$\Rightarrow \angle PQR + 120° = 180°$ [Linear pair]

$\Rightarrow \angle PQR = 180° - 120° = 60°$

$\therefore \angle S = \angle PQR$ [Opposite angles]

$\Rightarrow \angle b = 60°$

$\Rightarrow \angle a = \angle PQR$ [Corresponding angles]

$\Rightarrow \angle a = 60°$

Now, $\angle PSR + \angle SRQ = 180°$ [Adjacent angles]

$\Rightarrow 60° + \angle SRQ = 180°$

$\Rightarrow \angle SRQ = 180° - 60° = 120°$

$\Rightarrow \angle c = 120°$

Hence, $\angle a = 60°$, $\angle b = 60°$ and $\angle c = 120°$.

Multiple Choice Questions

1. Bisectors of adjacent angles of a parallelogram form an angle of 90°. Hence, the correct option is (d).

2. The sum of angles of an sides polygon is $(n - 2) \times 180°$ or $(2n - 4)$ right angles. Hence, the correct option is (c).

3. Diagonals of a kite do not bisect each other. Hence, the correct option is (b).

4. Sum of interior angles of a 11-sides polygon = $(11 - 2) \times 180° = 9 \times 180° = 1620°$.
Hence, the correct option is (a).

5. See Answers given in the book.

6. The sum of the exterior angles of a polygon is 360°, i.e., 4 right angles. Hence, the correct option is (c).

7. Number of sides of a regular polygon with exterior angle 45° = $360° \div 45° = 8$. Hence, the correct option is (b).

8. See Answers given in the book.

9. Exterior angle of a regular hexagon = $360° \div 6 = 60°$.

10. The opposite sides and all angles of a rectangle or square are equal. Hence, the correct option is (c).

Mental Maths

A. See Answers given in the book.

B. See Answers given in the book.

Higher Order Thinking Skills (HOTS)

1. In quadrilateral ABCD, by angle sum property,

$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow \angle A + 90° + 90° + 115° = 360°$

$\Rightarrow \angle A + 295° = 360°$

$\Rightarrow \angle A = 360° - 295° = 65°$

In quadrilateral ABFE,

$\Rightarrow \angle A + \angle B + \angle BFE + C = 360°$ [Angle sum property of a quadrilateral]

$\Rightarrow 65° + 90° + 85° + \angle C = 360°$

$\Rightarrow \angle C = 360° - 240° = 120°$

Now $\angle a + \angle c = 180°$ [Linear pair]

$\Rightarrow \angle a = 180° - 120° = 60°$

Also, $\angle b + 85° = 180°$ [Linear pair]

$\Rightarrow \angle b = 180° - 85° = 95°$

Hence, values of a, b and c are respectively 60°, 95° and 120°.

2. We know that the diagonals of a parallelogram bisect each other.

(i) From the given figure,

$5y + 2 = 6y - 3$

$5y - 6y = -3 - 2$

$-y = -5 \quad \Rightarrow \quad y = 5$

Also, $4x + 16 = 2x + 24$

$4x - 2x = 24 - 16$

$2x = 8 \quad \Rightarrow \quad x = 4$

(ii) From the given figure,

$2z - 6 = z - 1$ [Opposite sides are equal]

$2z - z = -1 + 6$

$z = 5$

$5x - 1 = 2x + 5$ [Diagonals bisect each other]

$5x - 2x = 5 + 1$

$3x = 6 \quad \Rightarrow \quad x = 6 \div 3 = 2$

Now $8y + 5 = 6y + 11$

$8y - 6y = 11 - 5$

$2y = 6 \quad \Rightarrow \quad y = 6 \div 2 = 3$

Hence, $x = 2$, $y = 3$ and $z = 5$

3. (i) The sum of 4 acute angles is less than 360°.
Hence, a quadrilateral cannot have all four angles acute.

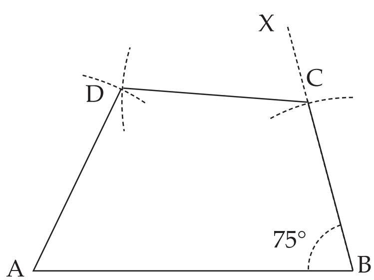
(ii) The sum of 4 obtuse angles is greater than 360°.
Hence, it is not possible to have a quadrilateral with all four obtuse angles.

13 Constructions

Exercise 13.1

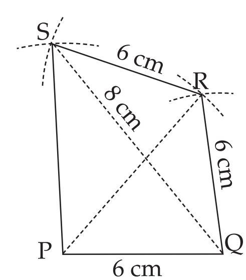
1. **Steps of construction:**
   - Draw a line segment AB = 6 cm.
   - At B, draw ∠ABX = 75°.
   - With B as centre and radius 6.5 cm, draw an arc cutting BX at C.
   - With C as centre and radius 7 cm, draw an arc.
   - With A as centre and radius 7.5 cm, draw an arc cutting the arc drawn in step 4 at a time D.

   Thus, ABCD is the required quadrilateral.



2. Similar work to be done as Q.1.
3. **Steps of construction:**
   - Draw a line segment PQ = 6 cm.
   - With Q as centre and radius 6 cm, draw an arc.
   - With P as centre and radius 8 cm, draw an arc cutting the arc drawn in step 2 at a point R.
   - With R as centre and radius 6 cm, draw an arc.
   - With Q as centre and radius 6 cm, draw an arc cutting the arc drawn in step 4 at a point S.

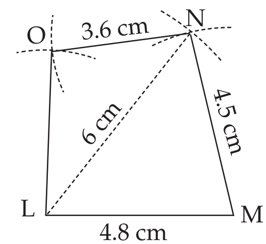
   Thus, PQRS is the required quadrilateral.



4. Similar work to be done as Q.3.

5. Similar work to be done as Q.1.
6. Similar work to be done as Q.3.
7. Similar work to be done as Q.3.
8. **Steps of construction:**
   - Draw a line segment LM = 4.8 cm.
   - With M as centre and radius 4.5 cm, draw an arc.
   - With L as centre and radius 6 cm, draw an arc cutting the arc drawn in step 2 at a point N.
   - With N as centre and radius 3.6 cm, draw an arc.
   - With L as centre and radius 4.2 cm, draw an arc cutting the arc drawn in step 4 at a point O.

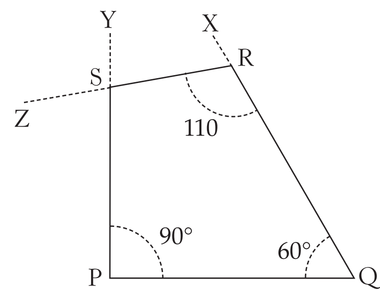
   Thus, LMNO is the required quadrilateral.



Exercise 13.2

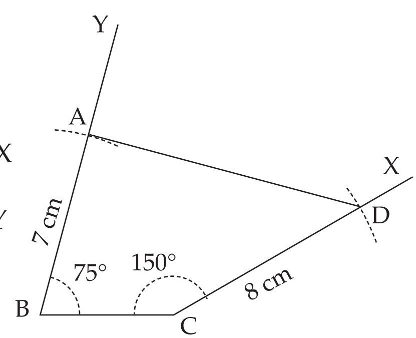
1. **Steps of construction:**
   - Draw a line segment PQ = 7 cm.
   - At P, draw ∠QPY = 90°.
   - At Q, draw ∠PQX = 60°.
   - With Q as centre and radius 6 cm, draw an arc cutting QX at R.
   - At R, draw ∠QRZ = 110° in which RZ cuts PY at a point S.

   Thus, PQRS is the required quadrilateral.



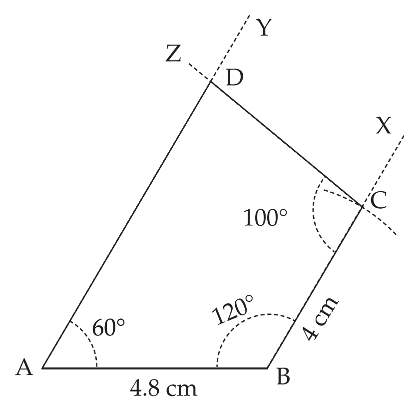
2. **Steps of construction:**
   - Draw a line segment BC = 5 cm.
   - At B, draw ∠CBY = 75°.
   - At C, draw ∠BCX = 150°.
   - With C as centre and radius 8 cm, draw an arc cutting CX at D.
   - With B as centre and radius 7 cm, draw an arc cutting BY at A.
   - Join A and D.

   Thus, ABCD is the required quadrilateral.



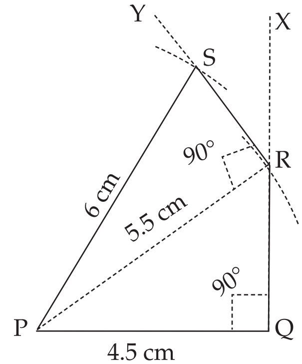
3. See **Answers** given in the book.
4. Similar work to be done as Q.2 of this Exercise.
5. Similar work to be done as Q.2 of this Exercise.
6. **Steps of construction:**
   - Draw a line segment AB = 4.8 cm.
   - At B, draw ∠ABX = 120°.
   - At A, draw ∠BAY = 60°.
   - With B as centre and radius 4 cm, draw an arc cutting BX at C.
   - At C, draw ∠BCZ = 100° which cuts AY at D.
   - Join C and D.

   Thus, ABCD is the required quadrilateral.



7. Similar work to be done as Q.2 of this Exercise.
8. **Steps of construction:**
   - Draw a line segment PQ = 4.5 cm.
   - At Q, draw ∠PQX = 90°.
   - At R, draw ∠PRY = 90°.
   - With P as centre and radius 5.5 cm, draw an arc cutting QX at R.
   - At R, draw ∠PRY = 90°.
   - With P as centre and radius 6 cm, draw an arc cutting RY at S.
   - Join R and S.

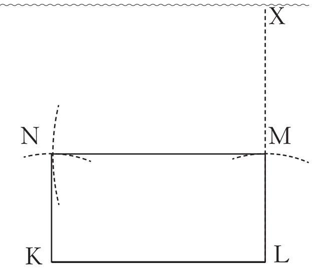
   Thus, PQRS is the required quadrilateral.



Exercise 13.3

1. **Steps of construction:**
   - Draw a line segment KL = 8 cm.
   - At K, draw ∠LKX = 90°.
   - With L as centre and radius 4 cm, draw an arc cutting LX at M.
   - With M as centre and radius 8 cm, draw an arc.
   - With K as centre and radius 4 cm, draw another arc cutting the arc drawn above at a point N.
   - Join MN and KN.

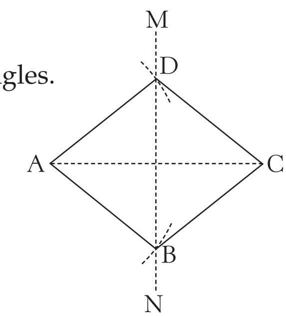
   Thus, KLMN is the required rectangle.



2. We know that the diagonals of a rhombus bisect each other at right angles.

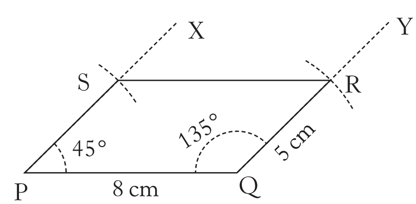
   Steps of construction:
   - Draw a line segment AC = 8 cm.
   - Draw the perpendicular bisector MN of AB.
   - With A as centre and radius 5 cm, draw arcs cutting MN at two points B and D.
   - Join AB, BC, CD and AD.

   Thus, ABCD is the required rhombus.



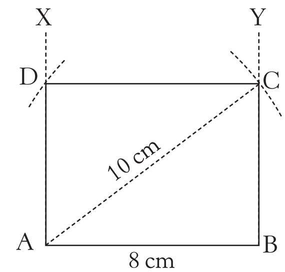
3. We know that the diagonals of a square are equal and bisect each other at right angles. So follow the same steps as in Q.2 above.
4. **Steps of construction:**
   - Draw a line segment PQ = 8 cm.
   - At P, draw ∠QPX = 45°.
   - At Q, draw ∠PQY = 180° – 45° = 135°.
   - With Q as centre and radius 5 cm, draw arcs cutting QY at a point R.
   - With P as centre and radius 5 cm, draw arcs cutting PX at a point S.
   - Join R and S.

   Thus, PQRS is the required parallelogram.



5. **Steps of construction:**
   - Draw a line segment AB = 8 cm.
   - At A, draw ∠BAX = 90°.
   - At B, draw ∠ABY = 90°.
   - With A as centre and radius 10 cm, draw an arc cutting BY at a point C.
   - With B as centre and radius 10 cm, draw an arc cutting AX at a point D.
   - Join C and D.

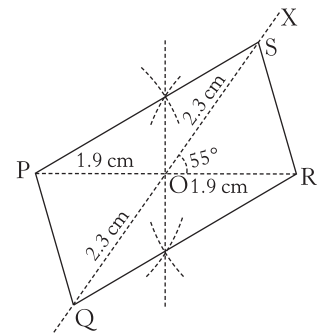
   Thus, ABCD is the required rectangle.



6. Similar work to be done as Q.2 of this Exercise.
7. We know that the diagonals of a parallelogram bisect each other.

   Steps of construction:
   - Draw a line segment PR = 4 cm.
   - Draw the bisector of PR which cuts PR at a point O.
   - At O, draw ∠ROX = 55°.
   - Set off OS and OQ half of 4.6 cm.
   - Join PQ, PS, SR and QR

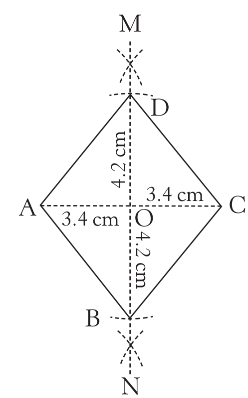
   Thus, PQRS is the required parallelogram.



8. We know that the diagonals of a rhombus bisect each other at right angles.

   Steps of construction:
   - Draw a line segment AC = 6.8 cm.
   - Draw the perpendicular bisector MN of AC which intersects it at a point O.
   - With O as centre and radius half of 8.4 cm, i.e., 4.2 cm, draw arcs cutting OM at B and ON at D.
   - Join AB, BC, CD and AD.

   Thus, ABCD is the required rhombus.



9. Similar work to be done as Q.4 of this Exercise.

Revision Exercise

1. See **Answers** given in the book.
2. See **Answers** given in the book.
3. Similar work to be done as Q.1 of Exercise 13.3.
4. Similar work to be done as Q.2 of Exercise 13.2.
5. By mistake this question is wrong, so ignore it.
6. Similar work to be done as Q.1 of Exercise 13.3.
7. Similar work to be done as Q.5 of Exercise 13.2.

8. **Steps of construction:**
   - Draw a line segment PQ = 8 cm.
   - With P as centre and radius half of 6 cm draw an arc.
   - With Q as centre and radius half of 5 cm draw another arc cutting the previous arc at a point R
   - With P as centre and radius half of 8 cm draw an arc.
   - With R as centre and radius half of 5 cm draw another arc cutting the previous arc at a point S
   - Join PS and RS.

   Thus, PQRS is the required rhombus.
9. Similar work to be done as Q.9 of Exercise 13.3.
10. Similar work to be done as Q.3 of Exercise 13.3.

Multiple Choice Questions

See **Answers** given in the book.

Mental Maths

A. See **Answers** given in the book.

B. See **Answers** given in the book.

Higher Order Thinking Skills

See **Answers** given in the book.

14 Visualising Solid Shapes

Exercise 14.1

1. See **Answers** given in the book.
2. The students will do it themselves.

Exercise 14.2

1.

	Solid shape	No. of faces	No. of Edges
(a)	Hexagonal prism	8	18
(b)	Square prism	6	12
(c)	Cuboid	6	12
(d)	Rectangular pyramid	5	8
(e)	Tetrahedron	4	6

2. The students will do it themselves.
3. (a) A hexagonal prism has 8 faces, 18 edges and 12 vertices.

Euler's formula is: V + F – E = 2

LHS = V + F – E = 12 + 8 – 18 = 20 – 18 = 2 = RHS

Hence, proved.

(b) A square prism has 6 faces, 12 edges and 8 vertices.

Euler's formula is: V + F – E = 2

LHS = V + F – E = 6 + 8 – 12 = 14 – 12 = 2 = RHS

Hence, proved.

(c) The given nested solid has 12 edges, 8 vertices and 6 faces.

Euler's formula is : V + F – E = 2

LHS = V + F – E = 8 + 6 – 12 = 14 – 12 = 2 = RHS

(d) Similar work to be done.

4.

Faces	6	A	5	C	4
Edges	E	15	B	30	6
Vertices	8	10	6	12	D
	E = V + F – 2 = 8 + 6 – 2 = 12	V + F – E = 2 = A = E + 2 – V = 15 + 2 – 10 = 7	B = V + F – 2 = 6 + 5 – 2 = 11 – 2 = P	C = E + 2 – V = 30 + 2 – 12 = 20	D = E + 2 – F = 6 + 2 – 4 = 8 – 4 = 4

5. (a) See Answers given in the book.
(b) See Answers given in the book.
(c) See Answers given in the book.

6. The students will do it themselves.

7. The students will do it themselves.

8. The students will do it themselves.

9. (a) The given net is of a hexagonal pyramid.
(b) The given net is of a tetrahedron

Revision Exercise

1. See Answers given in the book.

2. See Answers given in the book.

3. Euler's formula states the relationship between the number of faces, edges and vertices of a polyhedron which is as :
V + F – E = 2
Where V = number of vertices, F = number of faces and E = number of edges.

4. (i) A square has 2 faces, 4 edges and 4 vertices
Euler's formula is V + F – E = 2
LHS = 4 + 2 – 4 = 2 = RHS
Hence, proved.

(ii) A square has 6 faces, 8 vertices and 12 edges.
Euler's formula is : $V + F - E = 2$
$LHS = V + F - E = 8 + 6 - 12 = 14 - 12 = 2 = RHS$
Hence, proved.

(iii) Similar work to be done.

(iv) Similar work to be done.

5. Number of faces (F) of the polyhedron = 4 and its vertices (V) = 4
By using Euler's formula
$V + F - E = 2$
$\Rightarrow V + F - 2 = E$
$\Rightarrow 4 + 4 - 2 = E$
$\Rightarrow E = 6$
Hence, the polyhedron has 6 edges.

6. In a triangular prism :
$F = 5$, $V = 6$ and $E = 9$
Euler's formula is :$V + F - E = 2$
$LHS = V + F - E = 6 + 5 - 9 = 11 - 9 = 2 = RHS$
Hence, verified

7. Students will do it themselves.

8. See Answers given in the book.

9. In the given polyhedron, $V = 16$ and $E = 30$
By using Euler's formula :
$V + F - E = 2$
$\Rightarrow 16 + F - 30 = 2$
$\Rightarrow F = 32 - 16 = 16.$
Hence, the given polyhedron has 16 faces.

10. The students will do it themselves.

Multiple Choice Questions

1. A prism whose base is a polygon of n sides, then the prism has n + 2 faces.
Hence, the correct option is (a).

2. An octahedron has 6 vertices. Hence, the correct option is (a).

3. Using the Euler's formula :
$V + F - E = 2$
$\Rightarrow 8 + 6 - E = 2$
$\Rightarrow -E = 2 = 14 = -2 \qquad \Rightarrow E = 12$
Hence, the correct option is (c).

4. The given net is of triangular prism. Hence, the correct option is (c).

5. We know the sum of the numbers on opposite sides of a die is 7. Hence, the correct option is (a).

6. A tetrahedron is also known as tetrahedron. Hence, the correct option is (a).